VISCOUS DISSIPATION EFFECTS ON HEAT TRANSFER IN FLOW PAST A CONTINUOUSLY MOVING POROUS PLATE

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Summary: An Analysis of the effects of viscous dissipative heat on the temperature and rate of heat transfer in the flow past a continuously moving flat porous plate has been carried out. It has been observed that greater viscous dissipative heat causes a rise in the temperature and a fall in the rate of heat transfer for both suction and injection.

Introduction

Sikiadis [1] has shown that the flow past a continuously moving plate is different from the flow past a stationary plate studied by Blasius [2]. Tsou, Sparrow and Goldstein [3] studied the heat transfer aspect of continuously moving flat plate without considering viscous dissipative heat. But when the velocity of the plate is rather high in an incompressible fluid or when the Prandtl number, of an incompressible fluid is high, the viscous dissipative heat does play an important role in heat transfer problems. Hence, such a study was recently presented by Soundalgeker and Murty [4] in case of continuously moving flat plate.

The effects of suction and injection on the flow and heat transfer past a continuously moving porous plate were studied by Erickson, Fan and Fox [5] without taking into account viscous dissipative heat. It is now proposed to study the viscous dissipative heat effects on heat transfer in flow past a continuously moving porous plate. In Section 2, the mathematical analysis has been presented and in Section 3, the conclusions have been set out.

Mathematical Analysis

Here the x-axis is taken along the porous moving sheet in a stationary incompressible viscous fluid and the y-axis is taken normal to it. Then the flow and heat transfer of an incompressible viscous fluid under boundary layer approximation are governed by the following equation:

\[ \text{Mathematical Equation} \]
\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{1}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \nu \frac{\partial u}{\partial y} \tag{2}
\]

The boundary conditions are

\[
\begin{align*}
 u &= U_0, \quad v = v_0(x), \quad T = T_w \text{ at } y = 0 \\
 u &= 0, \quad v = 0, \quad T = T_\infty \text{ as } Y \to \infty
\end{align*} \tag{3}
\]

All the quantities have their usual meaning.

On introducing the following similarity transformations

\[
\eta = y \sqrt{\frac{U_0}{v x}}, \quad u = U_0 f', \quad v = \frac{1}{2} \sqrt{\frac{U_0}{x}} (\eta f' - f)
\]

\[
\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Pr = \frac{\mu c_p}{k}, \quad E = U_0^2 c_p (T_w - T_\infty)
\]

in equations (1) – (3), we get

\[
f'''' + \frac{1}{2} ff'' = 0 \tag{5}
\]

\[
\theta' + \frac{1}{2} Pr f \theta + Pr E f''^2 = 0 \tag{6}
\]

and the boundary conditions are

\[
\begin{align*}
 f(0) &= 2v_0(x) \sqrt{\frac{x}{v U_0}} = f_w, \quad f'(0) = 1, \quad \theta(0) = 1 \\
 f'(\infty) &= 0, \quad \theta(\infty) = 0
\end{align*} \tag{7}
\]

Here primes denote derivative with respect to \( \eta \). Also \( f_w > 0 \) indicates suction and \( f_w < 0 \) indicates injection. For \( E = 0 \), equations (5) and (6) subject to the boundary conditions (7) have been solved by Erickson et al. [5]. We have solved equations (5) and (6) under boundary conditions (7) numerically on a computer. Velocity profiles have been shown in Ref. [5]. In integrating energy, the fluid considered is air (\( Pr = 0.71 \)) and a fluid with \( Pr = 2 \). The temperature profiles are shown on Fig. 1 for \( E = 0, 0.01, 0.02 \) and for different values of suction and injection. We observe from Fig. 1 that both in the presence of suction and injection, presence of viscous dissipative heat causes a rise in the temperature. An increase in suction velocity leads to a fall in temperature whereas increase in injection

( 54 )
velocity leads to a rise in temperature. In the presence of both suction and injection, there is a fall in temperature due to increase in Prandtl number.

Knowing the temperature field, we now study the rate of heat transfer. In terms of Nusselt number, it can be expressed as

\[ \frac{Nu}{\sqrt{Re}} = f(0') \quad Nu = \frac{x q_w}{k(T_w - T_\infty)} \]

The numerical values of \( f(0') \) have been calculated and they are entered in Table I. We conclude from this table that both in the presence of suction and injection, there is a fall in the rate of heat transfer due to the presence of viscous dissipative heat. An increase in suction velocity leads to a rise in the rate of heat transfer and it decreases due to increasing injection velocity. The Nusselt number also increases with increasing the Prandtl number of the fluids.

Conclusions
1. An increase in \( E \) leads to a rise in the temperature of the fluid and a fall in the rate of heat transfer in the presence of both suction and injection.
2. An increase in suction velocity leads to a fall in temperature and a rise in the rate of heat transfer, whereas opposite is the case of injection effects.

Table I

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References

4. SOUNDALEKAR, V. M., AND RAMANA MURTY, T. V. : (To be published).

Table

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FIG. 1. TEMPERATURE PROFILES
FIG. 2. BOUNDARY LAYER ON A MOVING CONTINUOUS FLAT SURFACE WITH INJECTION