

SPECTRAL REPRESENTATION OF MEASURED SHALLOW WATER WAVES

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SYNOPSIS

Shallow water waves measured off Karwar, west coast of India during July 1988 at 16m water depth were used to make a comparative study on various theoretical wave spectral models. Selection of the best fit theoretical spectrum depends on the input parameters and local phenomena. In most of the cases the shallow water wave spectrum was derived by modifying the JONSWAP spectrum. The present study shows that the Scott and generalised spectra seem to yield a better representation of the measured shallow water waves. Based on deviation index analysis, both the Scott and Generalised spectra seem to represent the measured waves very well. However, Scott spectrum generally over-estimates energy values at peak frequency. The best-fit theoretical spectral formulations were found to vary with varying locations along the west coast of India.

INTRODUCTION

The shallow-water waves play an important role in governing the physical processes in the coastal areas. A representative shallow water wave spectrum has to be determined for application in the design of coastal structures. Interest in the formulation of an appropriate shallow water wave spectrum has increased in recent years.

A satisfactory model with universal applicability has not yet been evolved for representing shallow water wave spectral characteristics. A number of theoretical models have been attempted for different locations and climatic conditions. For a fully arisen sea condition, Pierson-Moskowitz spectrum (PIERSON-MOSKOWITZ, 1964) can be used to describe the spectral characteristics. For a growing sea state JONSWAP spectrum (HASSELMAN et al, 1973) can be used. DATTATRI et al (1977), NARASIMHAN and DEO (1979) have recommended the use of Scott spectrum (SCOTT, 1965) for waves in the coastal waters off Mangalore and Bombay High, west coast of India respectively. HARISH AND BABA (1986) proposed a new spectral form (PMK) combining the models of PIERSON-MOSKOWITZ and KITTAIGOROSDSKI et al (1975) for the south west coast of India. YOUNG (1992) reviewed on wave spectral representation and highlighted many limitations. Young made an attempt to define a consistent representation of the spectral parameters in terms of significant wave height and peak period. Shallow water wave spectral models are used to obtain an estimate of the entire wave spectrum from the known values of significant wave height and peak frequency, obtained by hindcast method or by direct measurement or by visual observation. To give an idea of the various factors which need to be taken into account, different spectral forms are used to compare the Fast Fourier Transform (FFT) spectra determined from measured waves off Karwar, west coast of India at 16m water depth.

THEORETICAL SPECTRA

Various theoretical spectra used for comparison with the FFT spectra of measured waves are i) Pierson-Moskowitz spectrum (PIERSON and MOSKOVITZ, 1964), ii) JONSWAP spectrum developed during the Joint North Sea Wave Project (HASSELMANN et al, 1973), iii) The Darbyshire spectrum (DARBYSHIRE, 1959), iv) The Pierson-Moskowitz-Kitaigorodskii (PMK) spectrum (HARISH and BABA, 1986),

v) The Wallops spectrum (HUANG et al, 1983), vi) TMA spectrum (BOUWS et al, 1985), vii) Scott spectrum (SCOTT, 1965) and viii) GLERL spectrum (LIU, 1983). The details of the spectrum which closely represent the present data are,

Scott spectrum

Scott (1965) modified the Darbyshire spectrum and adopted it for the British Towing Tank Panel standard wave spectrum as,

$$S(w) = 0.214 H^2 \exp\left\{-\left[\frac{(w-w_p)^2}{0.065(w-w_p+0.26)}\right]\right\}, \quad -0.26 < w-w_p < 1.65 \quad (1)$$

$$= 0, \quad \text{otherwise.}$$

Where,

H = significant wave height
w = angular wave frequency.

GLERL spectrum

The GLERL spectrum (LIU, 1983) is of the following generalised form,

$$S(f) = C_1 \left(\frac{m_0}{f_p}\right) \left(\frac{f}{f_p}\right)^{-c_2} \exp\left[-C_3 \left(\frac{f}{f_p}\right)^{(-c_2/C_3)}\right] \quad (2)$$

Where,

$$C_1 = [\exp(C_3) S(f_p) f_p] / m_0$$

$$C_1^{-1} = \exp[(1-C_3 + C_3/C_2) \ln C_3] \Gamma(C_3 - C_3/C_2) / C_2$$

$$(f_m/f_p)^2 = \exp[2C_3/C_2 \ln C_3] \Gamma(C_3 - 2C_3/C_2) / C_2 \Gamma(C_3 - C_3/C_2)$$

f_m = mean frequency

C_1 , C_2 & C_3 are solved by iteration technique.

RESULTS AND DISCUSSION

Characteristics of spectra

The shallow water waves measured off Karwar, West coast of India (Figure 1) during the monsoon period were used for the present study. A description of the spectral parameters in terms of local phenomena (wind speed, wind direction, water depth, etc.) would be valuable. Therefore, the spectral parameters were related to U_0/c_p , the ratio of the wind speed (U_0) at 10m above mean sea level in the direction of the waves at the peak of the spectrum to the phase speed (c_p) of those waves. The measured waves were classified under five ranges of U_0/c_p values, i.e., 0.0-0.1 to 0.4-0.5. Although spectra show a well defined peak at all ranges of U_0/c_p , in some cases low energy secondary peaks appear at higher frequencies showing the presence of both the sea and the swell. These secondary peaks seem to grow as the U_0/c_p values increase indicating the influence of direct wind input on the higher frequency components of the spectra. With the onset of monsoon the energy level at peak frequency increases. MANDAL and NAYAK (1988) reported that multipeakedness is a typical characteristics of the shallow water wave spectra. Multipeakedness seems to become predominant when the waves approach the coast. This is explained to be due to the presence of independent wave trains or the shifting of some energy from the main peak to higher frequencies due to refraction, shoaling and breaking phenomena. During the monsoon period, however, there mostly exists one major peak corresponding to the wind generated waves. The present study is directed to the wind driven waves which can be represented by a suitable theoretical wave spectrum.

Comparison with theoretical spectra

The deviation index (DI) proposed by LIU (1985) is used here to compare the measured spectra with various theoretical spectra. Table 1 gives the DI values for various models. As the local wind

speed increases, the PM and JONSWAP spectral peaks are found to increasingly deviate from the measured ones. For the location under study, it is evident that the PM and JONSWAP spectra do not represent the FFT spectra. Even though the wave energy is derived from the wind energy, the fact remains that the water has a much higher mass, one thousand times that of the air. Once the wind energy is transformed to the water in the form of waves, the water waves show a much longer memory on what has happened. As the waves propagate out of the storm area, they will still be high in spite of the local wind being low or high. DATTATRI et al (1977), and NARASIMHAN and DEO (1979) used Scott spectrum for wave in the shelf waters off Mangalore and Bombay High, west coast of India respectively. HARISH and BABA (1986) however, proposed a new spectral form called Pierson-Moskowitz-Kittaigorosdski (PMK) for the south west coast of India. The present study shows that the PMK model always over estimates spectral energies. However, the Scott spectrum seems to better represent the measured waves off Karwar, on the west coast of India.

The Scott and GLERL spectral models predict the energy reasonably well when compared to the other spectral models. Except GLERL spectral model, all other models are generally found to over-estimate the energy level at peak frequency region than that available in the measured spectra. All spectral models are found to predict the low energy part very well. Scott and GLERL spectra give the lowest values as shown in the table. Some typical FFT spectra are plotted and compared with the theoretical spectra (Scott and GLERL) as shown in Figures 2a-h. Even though the Scott spectrum is generally found to over-estimate the spectral energy at peak frequency, it closely represents the measured spectrum. The GLERL spectral estimates conform to the FFT spectral estimates obtained from the measured waves reasonably well. But the GLERL spectral model requires information on f_p , $S(f_p)$, f_m besides H_m and T_p . Liu (1987) proposed some empirical relations between these parameters based

on the measured data. The following empirical relations were derived for the location under study.

$$f_m = 0.82 f_p^{0.74} \quad (3)$$

$$s(f_p) = 20.64 (m_o)^{1.1} \quad (4)$$

Although the above relations are dimensionally incorrect, they are of practical value to provide some useful empirical spectral estimates when other parameters are not available.

CONCLUSIONS

Dynamics of shallow water waves are somewhat different from that of the deep water waves owing to many local phenomena such as water depth, reflection, refraction, shoaling etc. For deep water waves, well established theoretical wave spectra are available for various applications. The formulation of the theoretical wave spectrum varies at different shallow water locations owing to mainly local phenomena. A comparative study between wave spectra estimated by the FFT and various theoretical formulae indicates that the Scott and GLERL spectra closely represent the measured shallow water waves off Karwar on the west coast of India. Using the deviation index, both the Scott and GLERL spectra fit very close to the FFT spectra of measured waves. The present study also indicates that the theoretical spectral formulations vary at different locations in the west coast of India.

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Table 1 Deviation Index (DI)

Date	Month	Time	X2	X3	X4	X5	X6	X7	X8	X9
1	7	30	35.9	31.1	41.6	84.8	84.7	89.2	169.2	130.4
1	7	630	21.8	20.3	28.1	75.8	64.3	68.4	186.2	145.3
1	7	930	30.6	25.9	30.2	79.1	125.7	131.6	197.6	150.8
1	7	1230	20.6	19.9	36.5	75.7	38.6	38.6	180.0	135.1
1	7	2000	39.2	33.4	48.7	80.8	88.7	93.1	126.5	95.0
2	7	330	29.6	25.5	30.0	71.3	78.8	83.3	149.8	113.3
2	7	630	38.2	32.0	41.8	84.4	126.9	132.6	177.5	136.4
2	7	1230	25.0	22.2	29.9	73.4	166.2	173.3	151.3	111.5
2	7	1530	28.6	23.3	35.9	80.1	68.1	67.2	156.2	116.4
3	7	430	20.1	20.4	35.0	71.0	36.0	36.0	123.1	90.9
3	7	930	20.2	21.8	37.4	68.6	138.1	144.2	177.0	141.6
3	7	1230	17.5	16.4	24.9	70.1	88.9	93.7	130.5	98.7
3	7	1830	24.2	19.4	25.0	68.1	450.4	465.1	208.4	170.1
4	7	30	25.2	21.5	37.3	78.5	251.5	260.7	172.7	135.0
4	7	630	45.9	44.1	53.1	101.3	74.6	78.3	192.9	147.0
4	7	930	31.8	28.9	37.4	86.2	220.1	228.5	189.8	144.6
4	7	1830	16.2	18.6	35.3	67.5	388.3	401.3	150.4	114.2
4	7	2130	15.6	14.5	20.2	66.1	184.3	191.8	163.5	126.2
5	7	330	15.2	14.3	25.8	71.4	265.1	274.8	203.4	156.5
5	7	630	15.5	13.7	24.8	72.8	118.3	124.1	251.6	197.9
5	7	2230	13.8	15.5	29.1	76.3	79.7	84.4	319.9	254.1
6	7	130	13.4	14.3	21.0	74.5	116.8	122.5	284.1	221.2
6	7	730	24.3	25.5	35.6	82.0	296.3	306.8	326.2	261.3
6	7	930	13.8	18.0	34.3	75.5	206.1	214.2	225.7	173.4
6	7	1530	23.1	25.2	42.4	81.9	269.1	278.9	257.7	199.0
6	7	2230	20.7	21.3	29.6	70.6	106.8	112.3	237.8	189.9
7	7	730	13.9	13.6	26.3	77.7	138.6	145.0	284.0	223.8
7	7	1230	16.5	20.4	34.3	77.7	175.0	182.3	222.1	166.1
7	7	1830	19.6	18.9	33.7	85.1	149.1	155.7	319.3	251.3
8	7	130	16.6	16.2	31.4	83.5	245.1	254.3	275.8	208.2
8	7	430	16.6	19.2	27.2	79.7	142.3	148.7	281.1	211.0
8	7	930	24.2	28.2	44.5	83.0	119.8	125.5	248.9	185.6
8	7	1230	14.4	15.3	29.4	81.3	147.0	153.5	306.0	239.0
8	7	1530	17.5	15.1	29.8	81.7	138.0	144.2	296.3	228.5
9	7	430	19.3	21.4	32.7	79.6	113.2	118.8	268.5	209.3
9	7	930	24.1	26.2	37.6	76.9	64.7	69.0	281.2	224.2
9	7	1230	17.5	20.2	28.3	73.8	64.4	68.7	299.0	237.5
9	7	1530	15.4	18.7	30.7	75.3	65.0	69.3	313.3	246.5
9	7	1830	17.5	19.2	31.0	78.1	91.5	96.5	257.2	198.5
10	7	30	20.5	20.8	37.0	78.4	94.6	99.7	245.0	191.9
10	7	330	20.2	17.4	27.1	78.9	182.9	190.4	346.9	282.9
10	7	630	17.0	18.6	28.5	71.9	88.5	93.5	282.0	227.4
10	7	930	16.6	18.9	26.6	68.5	401.8	415.2	285.8	232.6
10	7	1530	26.7	34.2	44.3	67.5	134.6	140.9	259.4	204.3
10	7	1830	20.7	25.6	35.8	74.9	120.7	126.4	289.7	231.6
10	7	2230	25.9	28.9	45.9	84.8	53.6	53.6	251.5	194.1
11	7	130	20.7	24.4	43.4	80.0	35.4	33.5	226.7	173.3
11	7	430	28.6	25.1	41.9	95.0	107.0	112.0	233.9	178.0

X2 - GLERL
X3 - Scott
X4 - Darbyshire

X5 - Wallop
X6 - PM
X7 - JONSWAP

X8 - PMK
X9 - TMA

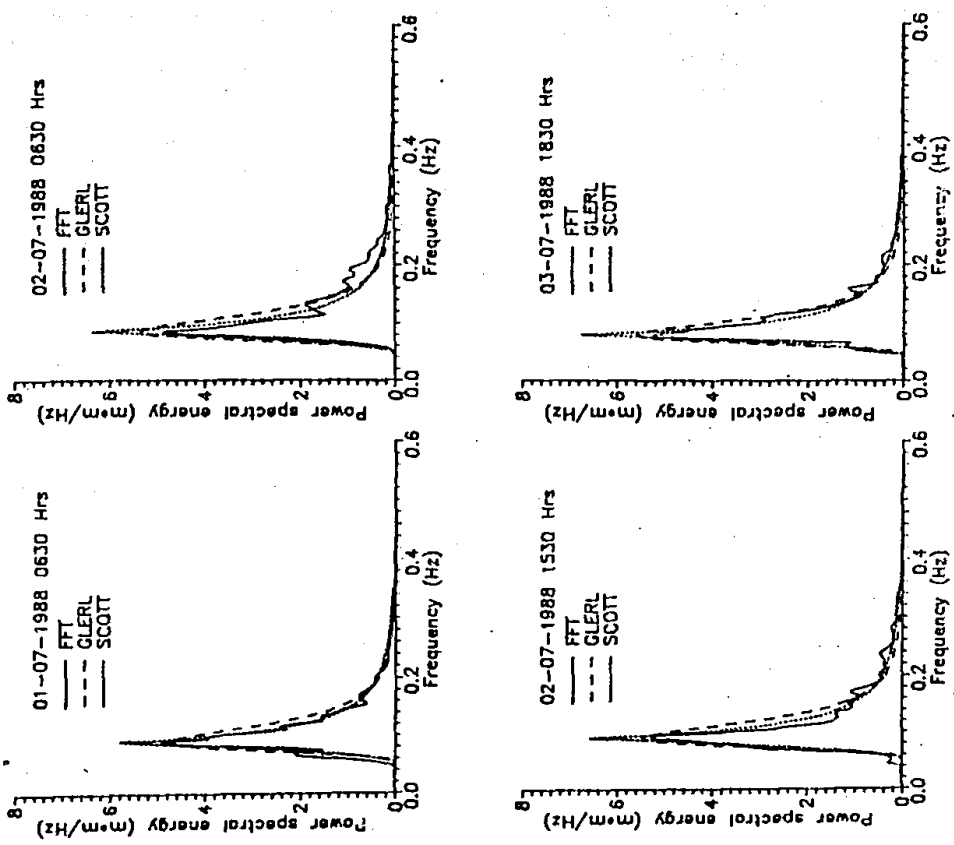


Fig. 2a. Comparison of FFT spectra with GLERL and Scott spectra

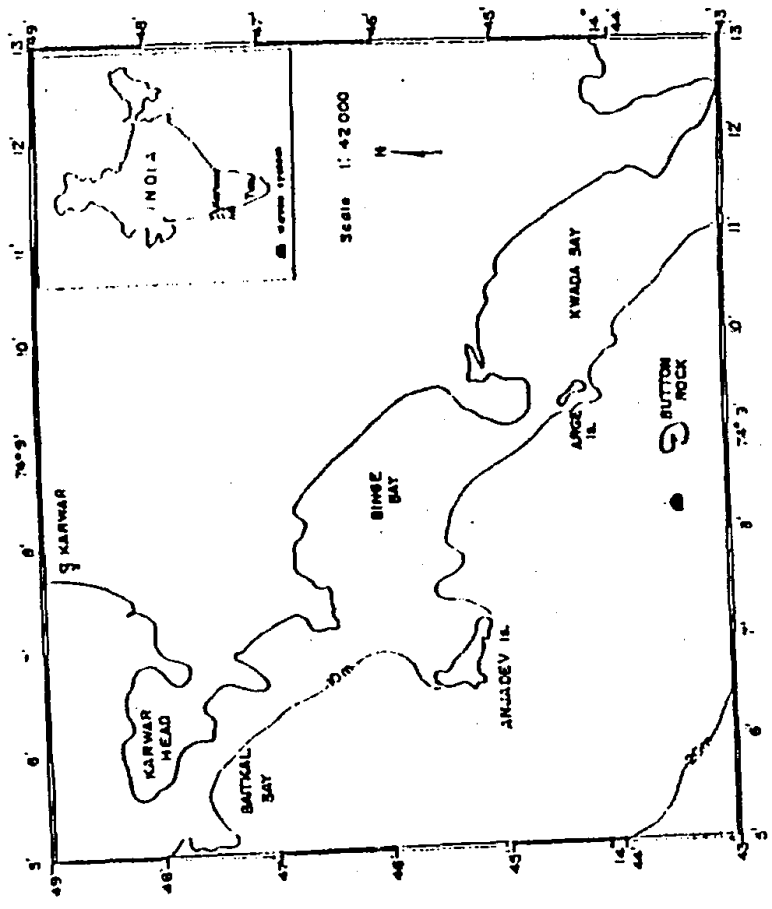


FIG.1. LOCATION MAP

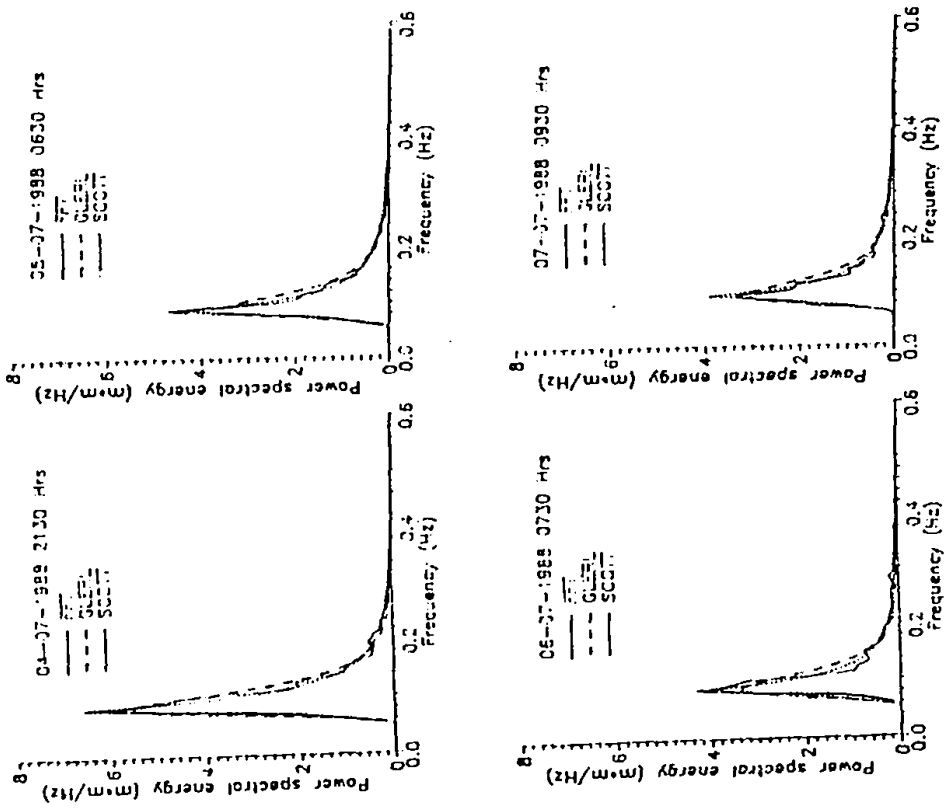


Fig. 2b. Comparison of FFT spectra with GLERL and Scott spectra

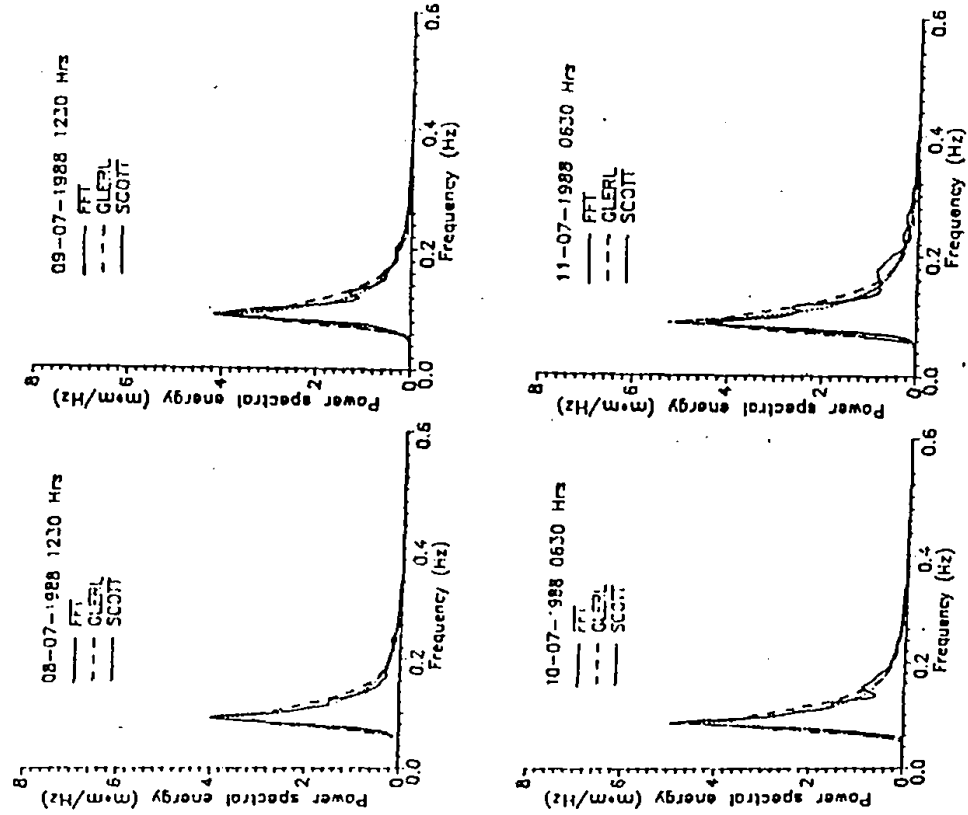


Fig. 2c. Comparison of FFT spectra with GLERL and Scott spectra