

SIGNIFICANCE OF RELATIVE VELOCITY IN DRAG FORCE OR DRAG POWER ESTIMATION FOR A TETHERED FLOAT

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ABSTRACT

There is difference in opinion regarding the use of relative velocity instead of particle velocity alone in the estimation of drag force or power. In the present study, a tethered spherical float which undergoes oscillatory motion in regular waves is considered for the analysis. Float velocity is computed through dynamical equation of motion and particle velocity using linear wave theory. The method of computation is briefly described. The results show that variation of particle velocity with respect to wave period, wave height or water depth is small compared to the variation of float velocity with respect to the same parameters. The results further indicate that the difference between float velocity and particle velocity is considerable and in such cases, relative velocity instead of particle velocity has to be considered for drag force or drag power computation. It is suggested that float velocity must be at least twice the particle velocity in order to use relative velocity in drag force or drag power estimation.

INTRODUCTION

The need for accurate estimation of hydrodynamic forces for the economic design of floating structures has created mathematical and experimental interest in recent years because of fast growing tendency of offshore industry. As exploration and exploitation of natural resources have been extended to deeper waters, the offshore industry has been forced to opt for compliant and floating structures. The wave-structure interaction is quite complex and very little is understood on the significance of relative velocity in drag force or drag power computation for a floating structure. Keeping this in view, the present study has been carried out to clearly understand the role of relative velocity in the estimation of drag force and drag power for a tethered spherical float oscillating in regular waves.

Garrison (1974 and 1975) has presented numerical results of hydrodynamic forces exerted on a floating sphere, describing harmonic oscillations under or on a free surface. Adeè and Martin (1974) treated the two-dimensional problem of moored floating objects in deep water using John's method. In all these investigations, only freely floating bodies are considered as an approximation to slackly moored conditions. Generally, drag force is ignored in the equation of motion for the following two reasons: (i) most of the offshore structures are large in size and drag force is almost negligible, and (ii) omission of drag force simplifies the solving procedure. Garrison (1978) developed his analysis using the method of distributed sources with allowances made for viscous effects when small diameter members give rise to flow separation. He stated that if viscous effects are disregarded, the results of the potential flow theory approach those based on the inertia term in the Morison equation. This shows that the drag term can be incorporated in the potential flow theory if found significant.

Even though the total fluid forces may not be affected significantly by viscous and vortex shedding effects, these may contribute appreciably to the damping, which is the major contributor for wave energy attenuation. Seymour and Isaacs (1974) and Agerton et al (1976) have studied the wave attenuation performance of the tethered float breakwater, assuming that the drag produced during the rapid oscillation of the floats is the major contributor for the dissipation of wave energy.

Since there is difference in opinion regarding the use of relative velocity instead of fluid particle velocity in estimating drag force for moving objects (Brebbia & Walker, 1979 and Vugts, 1979), in the present study both float and fluid particle velocities are individually found out to compare their magnitudes. Dunwoody and Vandiver (1987) are also of the opinion that if the structure as well as the fluid is in motion, particle velocity may be replaced by the relative velocity.

METHOD OF ANALYSIS

Ocean waves are random in nature. However, it has been shown by Milgram (1976) that the difference between the actual wave profile and the sinusoidal approximation is very marginal since most ocean waves have steepness less than 0.10. Hence, linear wave theory is used to compute fluid particle velocity. Horizontal particle velocity is given as,

$$u = \frac{\pi H \cosh k(z+d)}{T \sinh kd} \cos(kx - \omega t)$$

where, H = Wave height
 T = Wave period
 k = Wave number
 d = Water depth

The magnitude of particle velocity obtained at 16 equal intervals over a wave cycle is used for estimating average particle velocity. The suitability and satisfactory accuracy in adopting linear wave theory for practical applications have been confirmed by the studies of Hogben et al (1977). The float displacement and subsequently the float velocity are found out using potential flow theory, incorporating linearised drag force and added mass and damping coefficients determined from the motion generated velocity potential in the linear equation of motion (Vethamony, 1990).

For the analysis, a single tethered float which undergoes oscillatory motion in sinusoidal waves is considered. As the wave energy reduction by the float is mainly due to horizontal motion of the floats, only surge mode of motion is considered. If the fluid is assumed to be inviscid, incompressible and irrotational and the harmonic waves are of small amplitudes, then the surge motion of the body can be considered as harmonic with small amplitude. Therefore,

$$x(t) = \text{Re} \{ x_0 e^{-j\omega t} \}$$

Where,

$$\begin{aligned} x(t) &= \text{Surge displacement} \\ x_0 &= \text{Corresponding complex amplitude} \\ j &= \sqrt{-1} \\ \omega &= \text{Radian frequency} \\ \text{Re} &= \text{Real part} \end{aligned}$$

The simplified equation of motion for surge is written as,

$$\begin{aligned} \{ (m + \alpha)(-\omega^2) - (\beta + Q)j\omega + C \} x_0 \\ = f_w - (m + \alpha)\omega^2 \xi_0 - (\beta + Q)j\omega \xi_0 \end{aligned}$$

where,

$$\begin{aligned} m &= \text{Mass of the float} \\ \alpha &= \text{Added mass} \\ \beta &= \text{Damping coefficient} \\ Q &= \text{Drag term} \\ c &= \text{Restoring coefficient} \\ f_w &= \text{Amplitude of wave force} \\ \xi_0 &= \text{Amplitude of particle displacement} \end{aligned}$$

Therefore,

$$x_0 = \frac{f_w - (m + \alpha)\omega^2 \xi_0 - (\beta + Q)j\omega \xi_0}{\{ c - \omega^2(m + \alpha) - j\omega(\beta + Q) \}}$$

x_0 can be written as,

$$x_0 = x_{oc} + jx_{os}$$

where, x_{oc} and x_{os} are real functions.

The magnitude of x_0 , say α_0 can be obtained from,

$$|x_0| = \alpha_0 = \sqrt{x_{oc}^2 + x_{os}^2}$$

The computational method to determine the amplitude of float displacement is explained by Vethamony (1990). For harmonic motion, float velocity is written as,

$$\dot{x}(t) = \text{Re} \{ -j\omega x_0 e^{-j\omega t} \}$$

Therefore, amplitude of float velocity is

$$|\dot{x}| = \omega x_0 = \omega \alpha_0$$

Drag appears as a result of boundary layer separation and the formation of a wake behind the float.

Drag force (F_D) is given by,

$$F_D = 0.5 C_D \rho A |u|$$

where,

$$\begin{aligned} C_D &= \text{Drag coefficient} \\ \rho &= \text{Sea water density} \\ A &= \text{Frontal area of the float} \end{aligned}$$

As the float velocity is comparable or greater than the fluid particle velocity, it is advisable to replace the particle velocity term in the drag force with the relative velocity ($\dot{r} = \dot{x} - u$). Therefore,

$$F_D = 0.5 C_D \rho A |\dot{r}|$$

Relative velocity has got major influence on the calculation of structural response if the structure responds dynamically to the fluid excitation and this excitation is dominated by drag load (Vugts, 1979). To include drag force in the linear equation of motion, the non-linear term $|\dot{r}|$ is replaced with $C'\dot{r}$. Assuming that water particle motion and relative velocity are Gaussian processes, probability density function for the relative velocity can be obtained in terms of standard deviation of the relative velocity (σ_r). Then the linearised term, C' becomes $C' = \sqrt{\frac{8}{\pi}} \sigma_r$. Thus, only the standard deviation of relative velocity is needed to be known to linearise the drag force. Malhotra and Penzien (1970) have given a procedure (iteration method) to estimate the value of σ_r . The iteration is started with σ_u , the standard deviation of particle velocity and a cyclic procedure is used to obtain σ_r . Similarly, Kuchida et al (1986) expressed σ_r as,

$$\sigma_r = \sqrt{\sigma_x^2 + \sigma_u^2}$$

where,

$$\begin{aligned} \sigma_x^2 &= \text{Variance of structural velocity} \\ \sigma_u^2 &= \text{Variance of fluid particle velocity} \end{aligned}$$

Determination of σ_r involves iteration technique with an appropriately preassumed value for σ_x^2 . The method of Kuchida et al is not as accurate as that of Malhotra and Penzien

because of the difficulty in assuming an appropriate value for σ_i^2 . Using relative velocity drag force is written as,

$$F_D = 0.5 C_D \rho A \sqrt{\frac{8}{\pi}} \sigma_i (\dot{x} - u)$$

Seymour and Isaacs (1974), conducted experiments with widely differing wave climates and obtained σ_i between 40 and 80 cm/sec with a mean of 56 cm/sec. In the present study σ_i is considered as 0.56 m/sec. Theoretical results with $C_D = 0.6$ only are presented here.

Drag power (P_d) of the float is proportional to the cube of the relative velocity between float and fluid particles. Therefore,

$$P_d = Q (|\dot{x}| - u) (|\dot{x}| - u)^2$$

$$\text{where, } Q = 0.5 C_D \rho A \sqrt{\frac{8}{\pi}} \sigma_i^3$$

RESULTS AND DISCUSSION

The variation of float and fluid particle velocities with wave period, wave height, float size, water depth and depth of submergence of float are shown in Figs.1(a) to 1(e). The results show that variation of particle velocity with respect to the above parameters is very small compared to the variation of float velocity for the same parameters. The results further indicate that the difference between float and particle velocities is considerable and in such cases, relative velocity instead of particle velocity has to be considered for computing drag force or drag power. The float velocity increases with increase in wave period and wave height and diminishes with increase in float size. No variation in float velocity is observed with water depth or depth of submergence of float. Drag force also follows the same pattern (Figs.2(a) to 2(e)).

In general, when the size of an object increases, drag force also increases. However, in the present case, with increase in float size, float velocity considerably decreases and since particle velocity is very much less than the float velocity it causes reduction in the drag force. It appears that if the size of the float or floating structure is quite large, it is possible for the system to have less velocity than the particle velocity. This condition depicts that if the size of the floating structure is large compared to wave length, structural velocity is negligible and drag force could even be ignored, which would then be a diffraction problem. It is therefore suggested that float velocity must be at least twice the particle velocity in order to use relative velocity in drag force estimation.

Variation of drag power with respect to wave period, wave height, float size, water depth and depth of submergence of float is shown in Figs.3(c) to 3(e). As relative velocity increases with increase in wave period and wave height and decreases with increase in float size, drag power also follows the same pattern. No significant variation is found in drag power with changes in water depth, except for a slight variation at shallower depths where water particle velocity depends on water depth.

Drag power, which is the dissipative power of the float, is the most important factor useful for studying the wave attenuation characteristics of a group of tethered floats.

CONCLUSION

In the present study, only a spherical float is considered for studying the significance of relative velocity in drag force and drag power estimations. However, for more generalization, studies should be carried out with objects of different shapes.

APPENDIX : REFERENCES

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APPENDIX : NOTATION

- A = Frontal area of the float
- c = Restoring coefficient
- C_D = Drag coefficient
- F_D = Drag force
- f_w = Amplitude of wave force
- j = $\sqrt{-1}$
- m = Float mass

- Q = $0.5 C_D \rho A \sqrt{\frac{8}{\pi}} \sigma_i$
- \dot{r} = Relative velocity
- u = Horizontal particle velocity
- $x(t)$ = Float displacement
- x_0 = Amplitude of float displacement
- \dot{x} = Float velocity
- α = Added mass
- β = Damping coefficient
- ξ_0 = Amplitude of particle displacement
- ρ = Sea water density
- \dot{r} = Relative velocity between float and fluid particle
- ω = Radian frequency

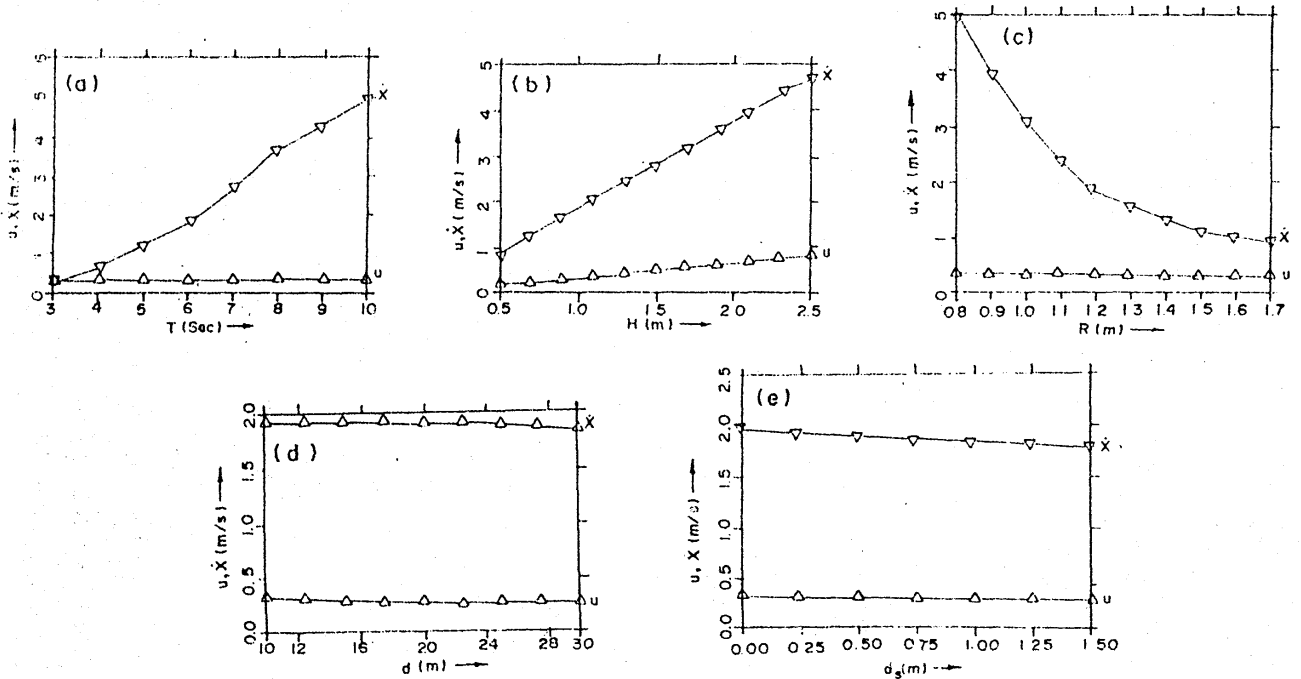


Fig. 1 Variation of particle velocity (u) and float velocity (\dot{x}) with (a) Wave period, (b) Wave height, (c) float size, (d) Water depth and (e) Depth of submergence of float

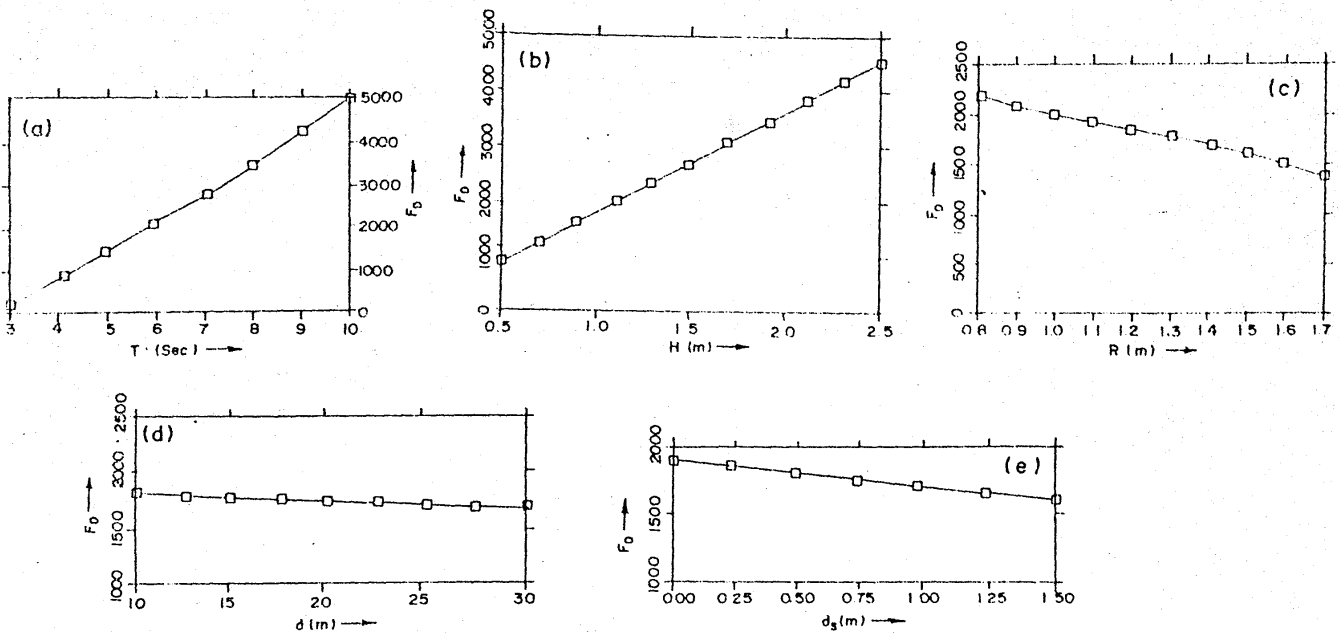


Fig. 2 Variation of drag force (F_D) with (a) Wave period, (b) Wave height, (c) Float size, (d) Water depth and (e) Depth of submergence of float

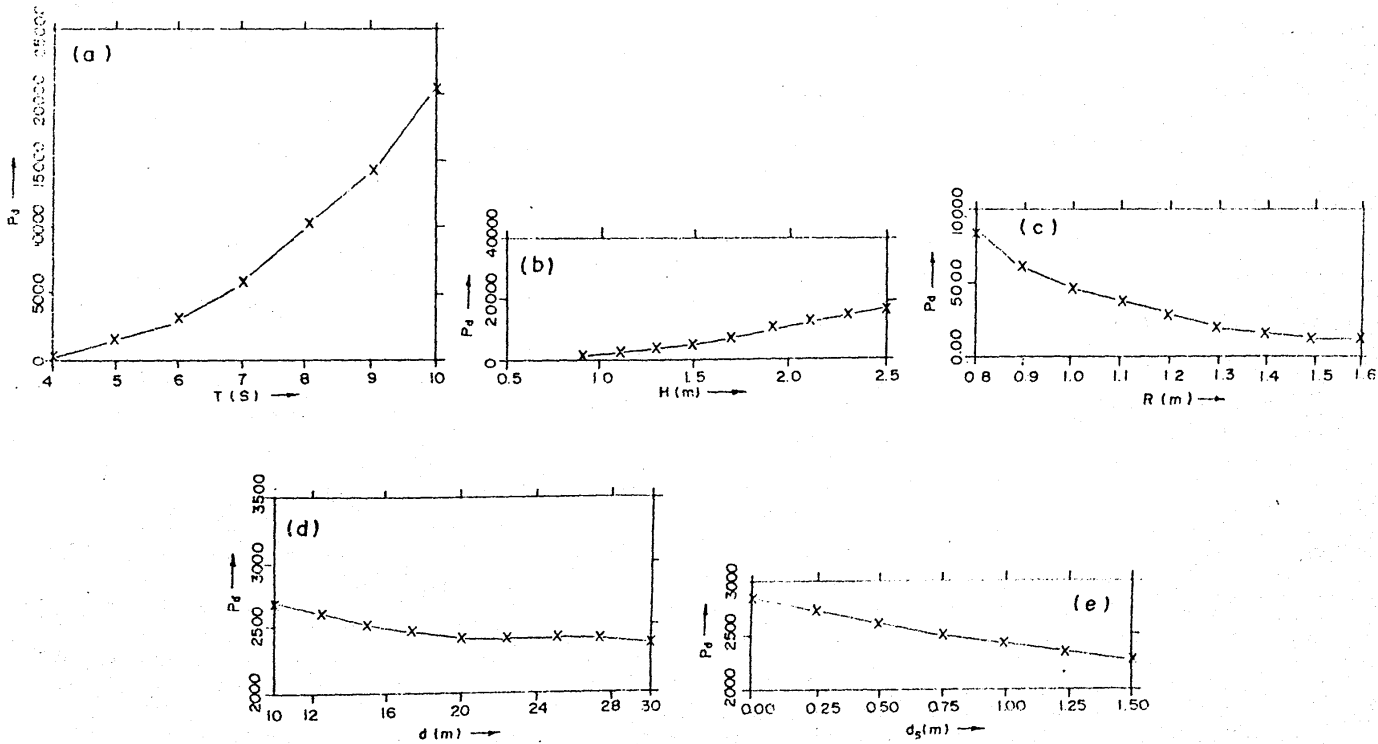


Fig. 3 Variation of drag power (P_D) with (a) Wave period (b) Wave height (c) Float size (d) Water depth and (e) Depth of submergence of float