COMPARISON OF BED SHEAR UNDER NON-BREAKING AND BREAKING SOLITARY WAVES

Jaya Kumar Seelam\textsuperscript{1} and Tom E. Baldock\textsuperscript{2}

Abstract: New experimental measurements of bed shear under solitary waves and solitary bores that represent tsunamis are presented. The total bed shear stress was measured directly using a shear cell apparatus. The solitary wave characteristics were measured using ultrasonic wave gauges and free stream velocities were measured using an Acoustic Doppler Velocimeter. The measurements were carried out in laminar and transitional flow regimes ($10^4 < \text{Re} < 10^5$). This sort of data is sparsely available in literature. In the absence of direct measurements, shear stress is indirectly estimated using velocity profiles or is inferred using standard friction factors. However, this indirect method has its limitations, e.g., under unsteady hydrodynamic conditions and relatively large roughness the assumptions of both approaches are no longer valid. More than 168 experimental runs comprising solitary waves and bores were carried out over a smooth flat bed with wave height to water depth ratio varying between 0.12 and 0.69. Analytical modeling was carried out to predict shear stresses using Fourier and convolution integration methods. This paper presents comparison of the measured and predicted bed shear stress and skin friction stress, together with estimates of traditional wave friction factors. Overall, the models can predict the bed shear stress with a satisfactory degree of accuracy.

Keywords: shear stress; friction factors; solitary waves; solitary bores; tsunami

1. INTRODUCTION

Depleting natural resources on the land has lead to an increased activity offshore for oil and gas exploration and exploitation around the globe. The search for such natural resources in farther and deeper oceanic regions necessitates understanding the effects of potential hazards on the seabed so as to understand the forces generated on subsea infrastructure. Tsunamis are one such potential hazard.
that has severe and direct effect on the subsea structures. The 2004 Indian Ocean tsunami not only devastated the coastal regions of Indonesia, India and other countries, but also increased awareness about tsunamis and there has been a surge in various aspects of research and development pertaining to the tsunami science. There were more than 15 significant tsunamis since the December 2004 Indian Ocean event till March 2010 and still there were casualties and devastation at many places, e.g., in Haiti, Chile, Indonesia, etc. Though the after effects of tsunami on the coastline are evident, the tsunami effect on the seabed and submarine structures during its propagation is not well documented in the literature. Potential impacts include the generation of currents and pressure gradients that are sufficient to mobilize significant quantities of sediment, trigger submarine landslides or locally liquefy submarine sediments. These impacts may be significant in terms of the stability of submarine pipelines and the foundations of other coastal infrastructure. In view of this, studies were initiated at the University of Queensland, to understand the effects of tsunami propagation on seabed and submarine features, especially continental slopes and submarine pipelines.

Solitary waves have been commonly used in the literature to represent tsunamis (Goring, 1979; Madsen et al., 2008; Synolakis and Bernard, 2006). Undular bores and solitary waves were observed during the December 2004 Indian Ocean tsunami event and these were simulated in a model study using the fully nonlinear dispersive method (FNDM) and the Korteweg-deVries (KdV) equation (Grue et al., 2008). Though the propagation of tsunamis can be modeled in near real time (Tang et al., 2009), details on bed shear stresses and pressure gradients that are exerted over the seabed are not well documented. Sediment transport modeling essentially requires bed shear stresses which is mostly derived indirectly from the velocity profiles (Nielsen, 1992) or from direct measurements. Bed shear stress in wave flumes or open channels have been directly measured using shear plates (Barnes and Baldock, 2007; Grass et al., 1995; Huo et al., 2007; Ippen et al., 1955; Riedel, 1972; You and Yin, 2007) or thermal techniques using hot film probes (Sumer et al., 1993). Other methods include estimation of shear stress from measurements of velocity profiles and estimate the shear stress using quadratic drag law (eqn.1) (Jensen et al., 1989; Liu et al., 2007).

2. METHODOLOGY

2.1 Laboratory and equipment setup

Laboratory measurements on bed shear stresses due to non-breaking solitary waves and breaking solitary bores were carried out in a tsunami wave flume. A piston wave maker, having a maximum stroke length of about 1.2 m, was used to generate these waves. The experimental setup consisted of a flat bed, a 1 in 10 slope and further flat bed region beyond the slope (Fig.1(a)). This setup is
considered to depict the ocean floor with the initial flat region corresponding to deep ocean floor, the slope representing continental slope and further flat bed representing the continental shelf. A shear plate apparatus placed flush with the flume bed is used to measure the bed shear stress. This apparatus shown in (Fig.1(b)), similar in design to that developed by University College London (Grass et al. 1995), consists of a Perspex casing cell with a steel smooth plate (1.21 mm thick, 100 mm long and 2500 mm wide, mass 226 g) supported on four thin tubular sway legs fixed to the bottom of the cell. The restoring force to the horizontal hydrodynamic forcing on the shear plate is provided by the tubular leg stiffness. A gap of 1 mm exists around the plate and the casing to accommodate the displacement of shear plate. The displacement of the shear plate is measured by an Indikon eddy-current proximity probe which resolves plate movement to 0.001 mm. The proximity probe measures the displacement of shear plate in both the directions thereby measuring both positive and negative shear stresses acting on the shear plate. Microsonic® ultrasonic wave gauges having a response time of 50 ms and capable of measuring water surface elevations up to ≤ 2% accuracy, were used to measure the water surface elevations. These ultrasonic gauges were calibrated by vertically moving the gauge by a known distance and recording the corresponding voltage. SONTEK® 2D ADV, with an accuracy of 1% of measured values for velocities < 1.7m/s, was used to measure the free stream velocities about 1 cm above the bed. High response DRUCK® pressure transducers with 0.15% accuracy of measured data were used to measure the pressures near the bed on either side of the shear plate. The distance between the wave paddle (with paddle at extreme forward position) and the shear plate is 8 m and the distance between the shear plate and the toe of the slope is 1.8 m. The length of the sloped region is 1.6 m with a further section of flat bed for more than 5 m. Experiments were carried out with the flume bed made of impermeable smooth marine plywood (hereinafter referred to as smooth bed) as well as with a sand paper affixed on to the bed to provide known roughness on the bed. In this paper only results pertaining to the smooth bed are presented.

2.2 Wave conditions and data acquisition
Solitary waves and bores were generated using two methods viz., (i) by utilizing the solitary wave generation method as per Goring and Raichlen (1980) and (ii) by using error functions to generate impulse waves as presented in Baldock et al. (2009). By varying the stoke length and the speed of the piston, solitary waves and bores of different amplitudes were generated. The typical wave paddle motion and resultant non-breaking solitary wave profiles generated is presented in Fig.2 whereas the paddle motion and resulting solitary bore profile is shown in Fig.3. For each of the wave or bore generated, bed shear stress, water surface displacement, velocity near the bed and pressure near the
bed were measured. In all 168 tests were conducted for the smooth bed case, wherein 100 tests are non-breaking solitary waves and 68 are breaking solitary bores. Data from the wave sensors, pressure sensors, and shear plate apparatus was acquired simultaneously at 50 Hz using National Instruments data acquisition system. Proprietary software of Sontek® ADV was used to acquire the velocity data. The acquisition of data from all the sensors and ADV were synchronized with the wave paddle software thereby obtaining synchronously. The noise from the data obtained from sensors was filtered using tools available in MATLAB® and this filtered data was used in further analysis.

2.3 Reynolds number estimation
For solitary waves, Reynolds number (R) is estimated using the velocity (u) obtained from wave period, length scale (A) and kinematic viscosity (ν) as $R = Au/ν$ (Sumer et al., 2010). The length scale (A) can be obtained from half stroke of water particle displacement which can be estimated from velocity and wave period. Although a wave period can be obtained for a non-breaking solitary wave by fitting a theoretical profile to the generated wave profile, it is not often possible to do the same for breaking solitary bores. While comparing both the non-breaking and breaking solitary waves, it would be appropriate to use identical method to estimating the Reynolds number. Therefore, in this study the length scale (A) is considered as the water particle semi-excursion and is obtained by integrating velocity from a minimum threshold value up to the maximum value.

2.4 Shear stress modeling and friction factors
Bed shear stress ($τ$) is often estimated using the quadratic drag law (eqn. 1) wherein the friction factor ($f$), density ($ρ$) and free stream velocity outside the boundary layer (or near bed velocity, $u_∞$) is used.

$$τ = \frac{1}{2} ρu_∞^2$$  \[1\]

Under strongly unsteady conditions, this method of estimating shear stress using a single friction factor fails since the shear stress is not in phase with the velocity. In view of this, direct measurement of shear stress or modeling of shear stresses using alternate methods is preferred. However, in the field it is not always possible to deploy a shear plate apparatus and therefore, models that predict shear stress reliably are needed. In this paper two methods were applied to predict the bed shear stresses and tested against the laboratory data. These methods are described in detail in Guard et al. (2010). One method applies a frequency response function to the free stream current,
applied in the frequency domain, and the other is based on a convolution integral of the free-stream acceleration, applied in the time domain. These methods are further described in this paper for sake of completeness.

For horizontally uniform flows and laminar boundary conditions, the laminar shear stress on top of the shear plate, utilizing Fourier transforms, can be calculated using eqn. 2.

\[ T(\omega) = \rho \sqrt{i \nu \omega} U_{\infty}(\omega) \]  

where, \( T(\omega) \) and \( U_{\infty}(\omega) \) are the Fourier transforms of \( \tau(t) \) and \( u_{\infty}(t) \), \( \omega \) is angular frequency, \( \nu \) is kinematic viscosity, \( \rho \) is fluid density, \( i \) is the imaginary unit and \( (\rho \sqrt{i \nu \omega}) \) is the frequency response function. In the case where only the surface elevations are available, the near bed velocity \( U_{\infty}(\omega) \) can be calculated using appropriate transfer function following linear wave theory as shown in eqn 3.

For waves following shallow water wave theory, the velocity can be obtained using eqn 4.

\[ U(\omega) = H(\omega) \frac{\omega}{\sinh(\omega h)} \]  

\[ U(\omega) = \sqrt{g \frac{d}{h}} H(\omega) \]  

The total force (\( \tau_T \)) (eqn. 5) measured by the shear apparatus is due to the shear stress (\( \tau_b \)) and the pressure gradient force generated by the free surface slope of the wave (e.g., Barnes et al., 2009; Grass et al., 1995; Riedel, 1972). The pressure gradient force (\( \tau_{pr} \)) can be estimated by estimating the equivalent force per unit area of the shear plate generated by the pressure gradient force (eqn. 6). If hydrostatic conditions are considered eqn. 6 reduces to eqn. 7 and for non-hydrostatic conditions, making use of the expression from linear wave theory the pressure force can be calculated as shown in eqn. 8.

\[ \tau_T = \tau_b + \tau_{pr} \]  

\[ \tau_{pr} = \frac{\partial p}{\partial x} \text{thickness of plate} \]  

\[ \tau_{pr} = \rho g \frac{\partial \eta}{\partial x} \text{thickness of plate} \]  

\[ P(\omega) = \rho gh \frac{H(\omega)}{\cosh(\omega h)} \]  

where, \( H(\omega) \) and \( P(\omega) \) are the Fourier transforms of surface displacement \( \eta(t) \) and pressure \( p(t) \).

The second method used to calculate the shear stresses in this study is by using convolution integrals on the acceleration in time domain. Liu and his coworkers (e.g., Liu and Orfila, 2004; Liu et al., 2006) showed that the shear stress can be obtained using eqn. 9. This method is a convolution
integration of the local acceleration weighted by the function \((t-t')^{-q}\). Torsvik and Liu (2007) describe techniques for efficient calculations of the convolution integral in eqn. 9.

\[
\tau(t) = \rho \sqrt{\nu'} \int_{0}^{t} \frac{\partial u_{w}}{\partial t'} dt' \quad [9]
\]

where \(\nu'\) is eddy viscosity and \(q\) is a parameter related to power function used in estimating the eddy viscosity as defined in Liu (2006). The parameter \(q=\frac{1}{2}\) corresponds to a constant \(\nu'\). In this paper two cases of eqn. 9 are used for the convolution method (i) using kinematic viscosity of water for \(\nu'\) with \(q=\frac{1}{2}\) and (ii) eddy viscosity defined by \(\nu' = k u^* z\) as in Nielsen (1992) but with \(q = 1/8\), where \(k\) is the von karman constant, \(u^*\) is the friction velocity and \(z\) is roughness height of the shear plate.

The pressure required to estimate the pressure gradient force is obtained using the following appropriate impulse response function as in eqn. 10. Since the wave height is measured in time domain, it can be transformed into spatial domain by assuming that the wave travels with constant speed thereby \(dt = \frac{dx}{\sqrt{gh}}\) and used in eqn. 10 to obtain pressures.

\[
p(t) = \rho gh + \int_{-3h}^{3h} \eta(x) \frac{\pi(x-x')}{2h} dx \quad [10]
\]

The frequency response and impulse response methods considered in this paper to estimate the shear stress do not involve use of empirical friction factors. However, since these are frequently used in practical applications, by using eqn. 1, friction factors can be calculated once the shear stresses and corresponding velocities are known. In this study both the total shear force measured by the shear plate and estimated bed shear stress are used to calculate the friction factors. The factors obtained from the total shear force are referred as Total force coefficients and those obtained from estimated shear stresses are referred as wave friction factors. Since the shear stress, velocity and the wave height vary with time, the estimated friction factors do vary with time. As common in the literature (e.g., Sumer et al., 2010), the maximum velocity is typically used to estimate the friction factor. However, the maximum shear stress need not occur at the time of maximum velocity, and in fact this is generally not the case. Therefore, the friction factors obtained using instantaneous velocities are also presented in order to compare with those obtained from maximum velocities. In order to compare the friction factors obtained from this study with other studies e.g., periodic waves, the derived friction factors are plotted on a wave friction factor diagram from Kamphuis (1978).

The time lag between the maximum values of velocity and maximum total shear stress, \(\tau_{r,\text{max}}\) and derived maximum skin shear stress, \(\tau_{\text{max}}\) are estimated by considering the difference between the
time of occurrence of maximum velocity and time of occurrence of maximum total shear stress. In order to convert this time lag in seconds to phase lag in degrees, a representative half wave period for the wave corresponding to $180^\circ$ is used. This representative half wave period is taken as the time elapsed between time of occurrence of a minimum cut off velocity (considered as 2.5% of the maximum velocity) and time of occurrence of maximum velocity in the forward direction.

3. RESULTS AND DISCUSSION

The typical time-series profile of a non-breaking solitary wave at leading and trailing edges of the shear plate apparatus, corresponding velocity, total shear force and derived bed shear stress profile are presented in Fig.4. A breaking solitary wave, or bore, profile and its corresponding parameters are shown in Fig.5. As illustrated in Fig.4, the total shear force changes sign to negative after passage of the wave crest over the measurement point, even though the velocity is positive over the entire wave propagation. This change in sign is attributed to the negative pressure gradient contribution during the deceleration phase of wave motion (Sumer et al., 2008). In case of solitary bores, the change in sign of total shear force is not as prominent and at times is absent. The bed shear stress (obtained by deducting the pressure gradient force from the total shear force) for non-breaking solitary waves is observed to lead the free stream velocity in general. In order to look at the differences between non-breaking solitary waves and bores maximum values obtained from each of the test cases are considered. The range of data for different parameters studied for both non-breaking and breaking wave conditions are given in Table 1.

Comparison of wave height to water depth ratio ($\gamma$) and Froude number shows that waves with $\gamma > 0.2$ deviate from shallow water linear wave theory and fall on a best fit line defined by $Fr = 0.625\gamma^{0.75}$. The data from the bores also fall on this best fit line (Fig.6). The relationship between $\gamma$ and maximum total shear stress ($\tau_{r,max}$) measured from the shear plate is shown in Fig.7. Both the non-breaking waves and bores show a linear trend, with non-breaking waves showing a best fit of $\tau_{r,max} = 3.5\gamma$ with $R^2$ of 0.995 whereas the bores fall on a best fit line represented by $\tau_{r,max} = 3.8\gamma$ with $R^2$ of 0.995. Fig.8 shows the comparison between measured maximum velocity and estimated maximum velocity obtained from shallow water wave theory (eqn. 4). Fig.9 shows similar comparison with velocities estimated from the linear transformation of wave height in frequency domain (eqn. 3), and improved accuracy.

Fig. 10 presents a comparison between measured maximum total shear stress and estimated total shear stress using the Fourier transformation method. The predictions for non-breaking waves and bores were found to be under estimated by about 14% but the bores had a larger scatter. Predictions
based on the convolution method with constant kinematic viscosity and parameter $q = ½$ showed that the predictions were well predicted for both non-breaking waves (0.3% under prediction) and bores (0.2% over prediction) (Fig.11). The predictions for the total shear stress are obtained from the convolution model with eddy viscosity using $\nu' = k u* z$ and with $q = 1/8$, wherein the predicted values for non-breaking waves were about 3% over predicted and about 4% over predicted for the bores (Fig.12). Consequently, overall, the total stress model performs very well over a full range of solitary waves and for severe breaking conditions.

Comparison between measured maximum skin shear stress and predicted skin shear stress using the Fourier transformation method showed that the predictions for non-breaking waves and bores were found to be under estimated by about 26% and 30% respectively (Fig. 13). The possible reason for FFT method underestimating the bed shear could be due to poor decomposition of solitary waves in frequency domain, where long flat surface is considered as a very long period wave, which could probably generate spurious frequencies. Predictions based on the convolution method with constant kinematic viscosity and parameter $q = ½$ showed that the predictions were under estimated by about 2% for both non-breaking waves and about 10% for bores (Fig.14). Predictions based on convolution model with eddy viscosity using $\nu' = k u* z$ and with $q = 1/8$ resulted in under estimation of the skin shear stresses by 5% for non-breaking waves and bores (Fig.15). The skin shear stress models based on convolution integration of the acceleration using kinematic viscosity and for laminar conditions were observed to perform well over a full range of solitary waves and for severe breaking conditions. The better performance of the model with laminar conditions ($q=1/2$ and using kinematic viscosity) could be due to the experimental data being in laminar regime where as the convolution model using $q=1/8$ and an eddy viscosity model varying with $z$ could be resulting in lower eddy viscosity values, thereby resulting in lower predicted shear stresses.

The total force factors estimated using the maximum velocity and the maximum total shear stress follow the same trend for both non-breaking and breaking waves (Fig.16). These force factors fall in the regime between smooth transition and lower limit of rough turbulent regime. The force factors estimated using instantaneous velocities at the time corresponding to maximum total shear stress indicate that the flow regime is on the lower limit of rough turbulent for non-breaking waves and in rough turbulent regime for the bores (Fig.17). This indicates that the force factors could vary by a factor of 2 in case instantaneous velocities are considered, which means the shear stresses could vary by a factor of 2 for a given Reynolds number. This is important while considering the sediment mobility wherein the sediments are subjected to a greater shear stress when the total shear force is considered.
Wave friction factors estimated using maximum velocity and maximum positive value of shear stress fall in the smooth-transition regime while the magnitude of the friction factors for both non-breaking waves as well as bores were of similar order (Fig.18). Wave friction factors estimated using instantaneous velocities provided comparatively similar estimates with respect to the values obtained using maximum velocity, however with similar order of magnitude (Fig.19). Thus, overall, the friction factors for these solitary waves and bores are similar to those obtained for periodic sine waves. However, it must be noted that because of phase differences between the velocity and bed stress, the full models are preferred over the simple application of the quadratic drag-law.

The phase difference between \( u_{\text{max}} \) and \( \tau_{\text{r, max}} \) for non-breaking waves and bores is presented in Fig.20 which shows that the phase difference for the bores is greater and more scattered compared to the non-breaking waves. Fig.21 shows the phase difference between \( u_{\text{max}} \) and \( \tau_{\text{max}} \) which indicates that the shear stress leads velocity for the non-breaking waves and for breaking waves or bores. The effect of this turbulence on the bed boundary layer will depend strongly on the water depth and wave-height to depth ratio, which may explain the larger scatter. In addition, estimates of phase differences are subject to noise and errors in estimating the pressure gradient force on the plate.

4. CONCLUSIONS
Solitary waves and solitary bores were generated in a tsunami wave flume and the direct bed shear stresses were measured in the laminar and transition regimes. The bed shear stress is observed to lead the free stream velocity both in non-breaking solitary waves as well as solitary bores. Analytical models were successfully applied to predict the shear stresses from the measured free stream velocities and the models used were found to predict the bed shear stresses reasonably. The friction factors derived from the skin shear stresses and free stream velocity indicate that the breaking solitary wave and the non-breaking waves do not exhibit distinct variations in the friction values, unlike the friction factors derived from the total shear stresses, indicating the dominant influence of pressure gradients on the total bed shear stress. The models for total bed shear stress are applicable to modeling the stability of submarine sediments. An increase in total force factors by two fold is possible in case of instantaneous velocities compared to maximum velocities, which could increase the sediment mobility. This is NIO contribution no._____.

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REFERENCES


Figure 1. (a) Experimental setup (b) Shear plate apparatus.
Figure 2. Non-breaking solitary waves (a) paddle displacement (b) wave profile.
Figure 3. Breaking solitary waves (a) paddle displacement (b) wave profile.
Figure 4. Typical measured parameters for non-breaking solitary wave. ____ velocity; **** total shear force; o o o o derived bed shear stress; ++++ predicted bed shear stress.
Figure 5. Typical measured parameters for breaking solitary wave. __ __ velocity; **** total shear force; o o o o derived bed shear stress; ++++ predicted bed shear stress.
Figure 6. Relationship between Froude number, $F_r = \frac{u_{max}}{\sqrt{gd}}$ and wave height to water depth ratio, $\gamma$. Solid line is linear wave theory ($u_{max} = \eta_{max} \sqrt{g/d} ; F_r = \gamma$); - - - - best fit ($F_r = 0.625 \ \gamma^{3/4}$). +++ - non-breaking waves; ooo – breaking waves.
Figure 7. Relationship between maximum measured total shear stress, $\tau_T$ and wave height to water depth ratio, $\gamma$. +++, non-breaking waves; ooo, breaking waves. ____ , best fit for non-breaking waves ($\tau_T = 3.5 \gamma$; $R^2 = 0.995$); - - - -, best fit for breaking waves ($\tau_T = 3.8 \gamma$; $R^2 = 0.995$).
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Figure 16. Total shear stress coefficients at maximum $\tau_f$ estimated using maximum velocity. Crosses correspond to non-breaking waves; Circles correspond to breaking waves; solid line - laminar solution (2/$\sqrt{\text{Re}}$); dashed line - lower limit of rough turbulent regime (8/$\sqrt{\text{Re}}$).
Figure 17. Total shear stress coefficients at maximum $r_\tau$ estimated using the instantaneous velocity. Crosses correspond to non-breaking waves; Circles correspond to breaking waves; solid line - laminar solution ($2/\sqrt{Re}$); dashed line - lower limit of rough turbulent regime ($8/\sqrt{Re}$).
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Figure 19. Wave friction factors at maximum $\tau$ estimated using instantaneous velocity plotted on wave friction factor diagram of Kamphuis (1978). Crosses correspond to non-breaking waves; Circles correspond to breaking waves; solid line - laminar solution ($2/\sqrt{Re}$); dashed line - lower limit of rough turbulent regime ($8/\sqrt{Re}$).
Figure 20. Phase differences between $u_{\text{max}}$ and measured $\tau_{T,\text{max}}$ (+++ non-breaking waves, median 53.26°) and (ooo breaking waves, median 62.31°)
Figure 21. Phase differences between $u_{\text{max}}$ and measured $r_{\text{max}}$ (+++ median 33.03° for non-breaking waves) and (ooo median 42.78° for breaking waves)
Table 1. Details of the range of experiments

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