

# On the Characteristics of Multi-peaked Spectra of Ocean Surface Waves

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*Multi-peaked ocean surface wave spectra obtained at a single location during a period of 30 hr in the Bay of Bengal are analysed. Major spectral peaks are identified and spectral energy has been assigned to each major peak and thereby significant wave height of independent wave trains is estimated. Significant wave height of the higher frequency region is found comparable to the other major peaks on several occasions. When the steepness and relative depth criteria are applied, it was found that these multi-peaked spectra are due to independent wave trains. The independent wave trains (in each spectrum) reveal well defined peak periods.*

## INTRODUCTION

For the design of offshore structures, ocean engineers derive the design wave parameters—significant wave height and significant wave period—from wave records or from climatic information. While forecasting ocean wave spectra at a single location, it is desirable to have a complete analysis of wave statistics over the region for long durations. When one examines the spectra of wind waves, multi-peaked spectra are often observed and are probably as frequent as those of single peaked spectra. The multi-peaked spectra may be due to independent wave trains arriving at a single point from different directions. A knowledge of characteristic features of the multi-peaked spectra may help the design engineer to develop better cost effective structures. It is possible that a few failures of offshore structures could have resulted due to the non-consideration of the multi-peaked spectral character of waves. In this paper, a study of surface wave characteristics of multi-peaked spectra is presented which were observed during the analysis of wave records collected onboard RV Gaveshani and an attempt has been made to assign the statistics for major peaks observed in the wave spectra.

## ANALYSIS OF DATA

During September-October 1980, RV Gaveshani collected several wave records, each of 15 min duration using the IOS shipborne wave recorder. The site is located off Godavari, Bay of Bengal at a depth of 500 m. During wave recording the ship remained stationary.

The data have been corrected for hydrodynamic attenuation due to the submergence of two sensors of the wave recorder which are installed 2 m below the sea level.

In all, 27 records have been analysed both by spectral and zero-upcrossing methods<sup>1</sup>. The power spectra have been computed through auto-correlation. Details of the methods employed and wave statistics obtained through both the methods are given<sup>2</sup>. Out of these 27 spectra, nine spectra (continuously recorded for 30 hr) showed multiple peaks (Fig 1) and an analysis of these is presented here.

## IDENTIFICATION OF MAJOR PEAKS AND ENERGY ASSIGNMENT

Wave records for the multi-peaked spectra (Fig 1) are collected on October 3-4, 1980 at intervals ranging between 2 hr and 5 hr. Following Thompson<sup>3</sup>, major peaks have been identified by the difference in energy density between the peak and the preceding spectral valley. If the difference is greater than 3% of the total energy density in the spectrum  $E_d$  [Fig 2 (a)], these peaks are considered significant and are denoted as major peaks in the spectra. A few peaks with  $E_d$  less than 3% were also noticed in some spectra. Table 1 gives a listing of the peaks along with  $E_d$  for major peaks. In some spectra, as can be seen from the figure, a sudden increase in energy density at higher frequencies comparable to that at lower frequencies is noticed. The attenuation correction factor is very large for higher frequencies and increases nearly exponentially

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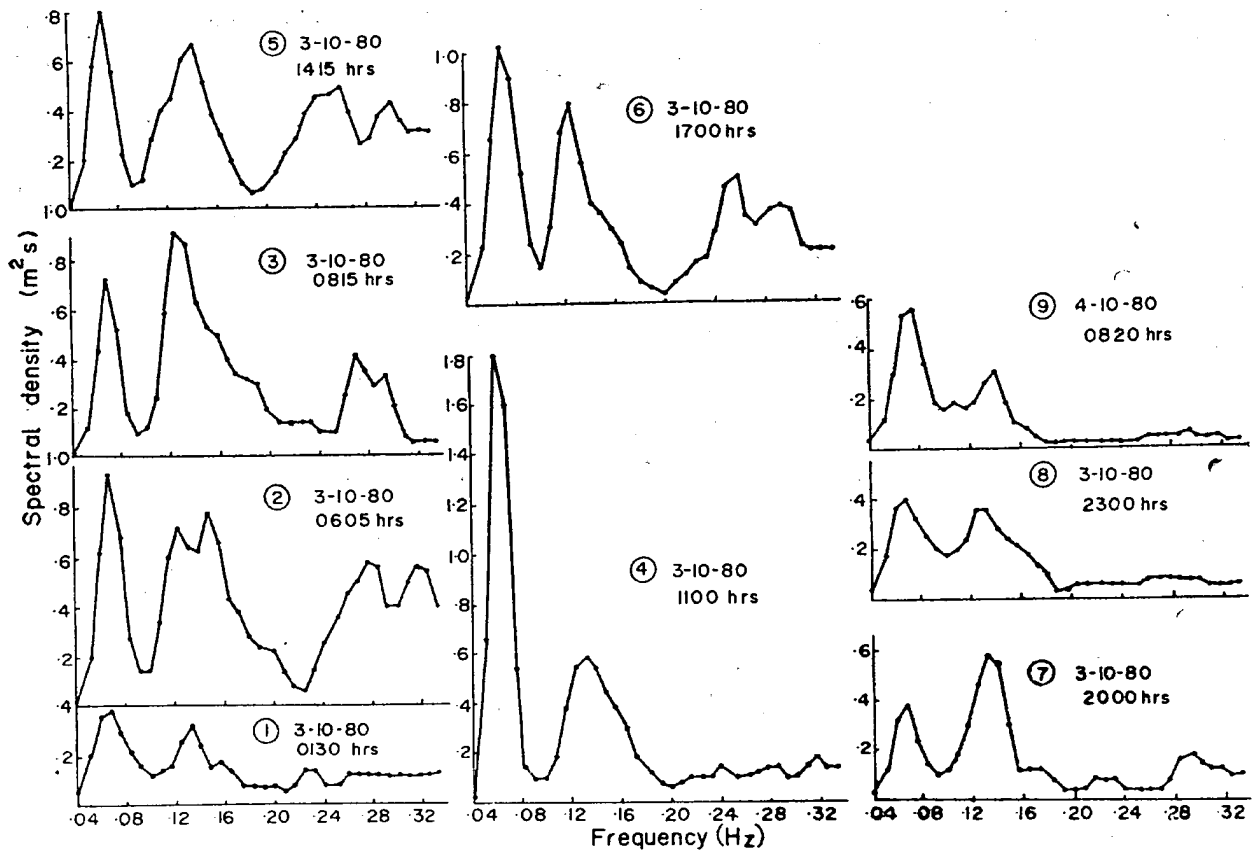


Fig 1 Multipieaked wave spectra

towards higher frequencies. The attenuation correction factor,  $C$ , of the shipborne wave recorder is given by

$$C = \{\omega^{-4} \exp(2.5D\omega^2/g)\} \left[ \{(\omega_1^2 - \omega^2)^2 + \alpha_1^2 \omega_1^2 \omega^2\} \times \{(\omega_2^2 - \omega^2)^2 + \alpha_2^2 \omega_2^2 \omega^2\} \right]^{\frac{1}{2}}$$

where  $\alpha_1 = 1.916$ ;  $\alpha_2 = 1.241$ ;  $\omega_1 = 0.09498$

rad/sec;  $\omega_2 = 0.1065$  rad/sec;  $\omega = \frac{2\pi}{T} \approx \frac{2\pi}{T_z}$ ;

$g = 9.81$  m/sec<sup>2</sup>;  $T$  = wave period;  $T_z$  = zero-crossing period;  $D$  = transducer depth in  $m = 2$  m for the NIO-RV Gaveshani SBWR.

TABLE I CLASSIFICATION OF PEAKS

SPECTRA NO	TOTAL NUMBER OF PEAKS	MAJOR PEAKS	$E_d$ FOR MAJOR PEAK (1)	$E_d$ FOR MAJOR PEAK (2)
1	2	2	5.8	3.7
2	3	2	6.2	3.9
3	3	2	6.4	7.5
4	2	2	15.0	4.5
5	4	2	6.0	4.6
6	4	2	8.1	5.5
7	2	2	5.4	7.8
8	2	2	6.3	3.3
9	2	2	11.6	3.0

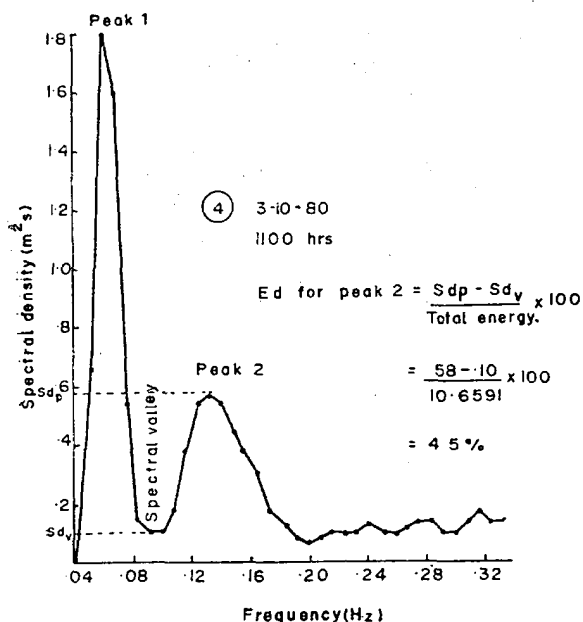
NB: Peaks in the higher frequency region, even if they exceed 3% of total, are not considered as major peaks.

Sufficient caution is necessary for interpreting these spectra at higher frequencies. Fig 2 (b) shows the procedure of assigning spectral energy to the peaks. The spectrum is partitioned at the lowest point between two successive peaks and the spectral energy ( $S_{p_2}$ ) within two partitions consisting of the peak is estimated as follows:

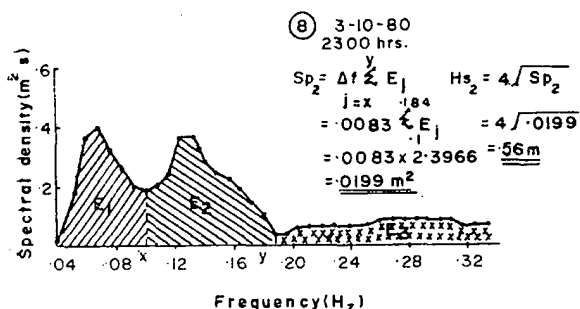
$$S_{p_2} = \Delta f \sum_{j=x}^y E_j$$

where  $S_{p_2}$  is the spectral energy of the peak 2,  $\Delta f$ , the frequency bandwidth ( $=0.0083$ ),  $E_j$ , the energy density between  $x$  and  $y$ , and  $x, y$ , the lower and upper bound frequencies of the peak 2 (Fig 2 (b),  $x = 0.1$  Hz,  $y = 0.184$  Hz).

Total energy density of the whole spectra ( $E$ ), spectral energy density of each of the peaks ( $E_1, E_2$ , etc) and that in the higher frequency region ( $E_H$ ) are given in Table 2. An examination of all the spectra in Fig 1 shows that there is considerable spectral energy in the high frequency zone (spectra 2, 3, 5 and 6) and is probably related to the locally generated seas. Inclusion of this spectral energy with the preceding peak as was done by Thompson<sup>3</sup> will overestimate the significant height of the wave train ( $H_{s_2}$  in Table 3) representing the preceding peak. If an estimate of the basic characteristics of wave trains approaching the point of observation is needed it would be necessary to treat the higher frequency portion of the energy separately.



(a)



(b)

Fig 2 (a) Illustration for  $E_2$ , and (b) illustration for spectral energy assignment

### ESTIMATION OF SIGNIFICANT WAVE HEIGHT AND SIGNIFICANCE OF PEAK FREQUENCY

The significant wave height,  $H_s$ , is normally determined using the relationship,  $H_s = 4 \sqrt{SMO}$ , where

SMO is the zeroth spectral moment. The significant height of the major peaks can be found out from the respective peak spectral energy assigned, using the formula  $H_{s_i} = \sqrt{4 S_{p_i}}$  where  $H_{s_i}$  is the significant wave height of the  $i$ th major peak representing the wave train and  $S_{p_i}$  the spectral energy of the  $i$ th major peak. Similar technique is applied to find out the significant height,  $H_T$ , of the higher frequency part. The estimates of  $H_s$ ,  $H_{s_i}$  and  $H_T$  are given in Table 3. For spectra 3, 4, 8 and 9,  $H_T$  is less than  $H_{s_1}$  and  $H_{s_2}$  and it shows that energy contribution of distant travelling independent swell trains are dominant over the local seas. On the contrary, for spectra 5 and 6,  $H_T$  is greater than  $H_{s_1}$  and  $H_{s_2}$  and it indicates the dominance of local seas over the swell trains represented by major peaks in the spectra. For example, see the contrast between the spectra 5 and 6 and 8 and 9 beyond the frequency of about 0.2 Hz. It is believed that the energy in the high frequency portion of the spectrum is primarily due to locally generated wind waves. An examination of various spectra and wind speeds<sup>4</sup> show that there exists strong correlation between higher frequency portion energy and wind speed.

Thompson's procedure overestimates  $H_{s_2}$  and the percent of overestimation is also shown in Table 3. The  $H_s$ , which takes into consideration the entire energy of the spectrum, and  $H_{s_p}$ , as estimated by taking

TABLE 2 SPECTRAL ENERGY DENSITY

SPECTRA No	Spectral Energy Density, $m^2/s$			
	TOTAL, $E$	PEAK 1, $E_1$	PEAK 2, $E_2$	HIGH FREQUENCY REGION, $E_H$
1	5.7310	1.8323	2.0351	1.8636
2	14.7337	2.8921	6.5049	5.3367
3	10.5955	2.1160	6.4889	1.9946
4	10.6591	4.8679	3.9234	1.8678
5	12.2553	2.8690	3.9630	5.4233
6	12.0872	3.7228	3.9659	4.3985
7	6.1415	1.3155	2.9521	1.8739
8	5.7253	1.9422	2.3966	1.3865
9	4.3023	2.5455	1.1935	0.5633

TABLE 3 VALUES OF SIGNIFICANT WAVE HEIGHTS

SPECTRA No	$H_s$ , m	$H_{s_1}$ , m	$H_{s_2}$ , m	$H_T$ , m	$H_{s_2T}$ , m	OE, %	$H_{sp} = \sqrt{H_{s_1}^2 + H_{s_2}^2 + H_T^2}$ , m	$H_{s_1}/H_{s_2}$
1	0.87	0.49	0.52	0.50	0.72	38	0.87	0.94
2	1.38	0.62	0.93	0.84	1.25	34	1.40	0.67
3	1.25	0.53	0.93	0.51	1.06	14	1.19	0.57
4	1.19	0.80	0.72	0.50	0.88	22	1.19	1.10
5	1.26	0.62	0.73	0.85	1.12	53	1.28	0.85
6	1.30	0.70	0.73	0.76	1.05	44	1.27	0.96
7	0.90	0.42	0.63	0.50	0.80	27	0.90	0.67
8	0.87	0.51	0.56	0.43	0.71	27	0.87	0.91
9	0.86	0.58	0.40	0.27	0.48	20	0.88	1.45

$H_{s_2T}$  = significant wave height of peak 2 with addition of energy from high frequency region (Thompson<sup>3</sup>)

$$OE = \frac{H_{s_2T} - H_{s_2}}{H_{s_1}} \times 100$$

the different peaks and higher frequency part energy, are also listed in Table 3, and the estimates are nearly the same. If multiple peaks are appearing due to individual swell trains it would not be prudent to add higher frequency energy to the preceding peak since this procedure overestimates significant wave height of the preceding peak.

Spectral peak frequency  $f_p$  of major peaks is directly obtained from the spectra and from that peak periods can be evaluated. It is found that peak periods  $T_{p1}$  and  $T_{p2}$  of each spectrum ranged between 17.1 sec and 13.3 sec and between 8.1 sec and 6.8 sec, respectively. If the natural frequency of the structure coincides with any of the peak frequencies, resonance occurs and the amplitude of motion of the structure may become remarkably large leading to the collapse of the structure. Thus, it is believed that the information on multi-peaked spectra would help for a better design.

### INDEPENDENT WAVE TRAINS

Major spectral peaks are often an indication of independent wave trains. In the present study, independent wave trains in each spectrum has well defined peak periods. The mean ratio of  $Hs_1$  and  $Hs_2$  is 0.9 while that for  $T_{p1}/T_{p2}$  varies from 1.85 to 2.41 (Table 4). The steepness and relative depth criteria<sup>3</sup> may be applied to the spectra to find the presence or otherwise of the independent wave trains. The coarse guidelines for independent wave trains to occur are

$$\text{steepness} \left( \frac{Hs_i}{g T_{pi}^2} \right) < 0.008$$

and

$$\text{relative depth} \left( \frac{d}{g T_{pi}^2} \right) > 0.01$$

where  $d$  is the station depth (= 500 m),  $T_{pi}$ , the peak periods of major peaks ( $i = 1, 2, \dots$ ), and  $Hs_i$ , the significant wave height of major peaks.

TABLE 4 PEAK PERIODS AND THEIR RATIOS

SPECTRA NO	$T_{p1}$ sec	$T_{p2}$ sec	$T_{p1}/T_{p2}$
1	15.0	7.4	2.02
2	15.0	6.8	2.21
3	16.1	8.0	2.01
4	17.1	7.1	2.41
5	15.0	6.9	2.17
6	15.0	8.1	1.85
7	15.4	7.5	2.05
8	15.0	7.4	2.03
9	13.3	7.1	1.87

TABLE 5 STEEPNESS ( $S$ ) AND RELATIVE DEPTH ( $d$ ) FOR PEAKS 1 AND 2

SPECTRA No	$S_1$	$S_2$	$d_1$	$d_2$
1	0.00022	0.0010	0.2268	0.9317
2	0.00028	0.0021	0.2268	0.1103
3	0.00021	0.0015	0.1968	0.7572
4	0.00028	0.0015	0.1745	0.1012
5	0.00028	0.0016	0.2268	0.1072
6	0.00032	0.0011	0.2268	0.7776
7	0.00018	0.0012	0.2151	0.8412
8	0.00023	0.0010	0.2268	0.9317
9	0.00033	0.0008	0.2884	0.1012

Table 5 shows these values. The steepness values of two peaks are found to be lower than 0.008 and relative depths are greater than 0.01, thereby showing that the peaks are due to independent wave trains.

### CONCLUSIONS

The analysis of wave records collected at a single location during a period of 30 hr yielded multi-peaked spectra with two major peaks in each spectrum. These multi-peaked spectra are due to independent wave trains having well defined peak periods. The inclusion of spectral energy in the higher frequency region with the preceding peak overestimates the significant height of the preceding peak representing the wave train. Hence, it is necessary to separate the higher frequency energy region from the preceding peak.

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