

Heat transfer in flow past a continuously moving porous flat plate with heat flux

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Abstract. The analysis of the heat transfer in flow past a continuously moving semi-infinite plate in the presence of suction/injection with heat flux has been presented. Similarity solutions have been derived and the resulting equations are integrated numerically. It has been observed that an increase in suction value leads to an increase in temperature whereas it is opposite in the case of injection.

Wärmeübergang in einer Strömung hinter einer wärmeabgebenden, kontinuierlich bewegten, porösen Platte

Zusammenfassung. Es wird eine Analyse des Wärmeübergangs in einer Strömung hinter einer bewegten, halbunendlichen Platte bei gleichzeitiger Absaugung bzw. Ausströmung präsentiert. Mit Hilfe von Ähnlichkeitsbetrachtungen wurden Lösungen erarbeitet und die daraus resultierenden Gleichungen wurden numerisch integriert. Als Ergebnis zeigte sich, daß zunehmende Absaugung zu einer Erhöhung der Temperatur führt, während bei der Ausströmung das Gegenteil der Fall ist.

1 Introduction

Boundary layer flows of a viscous, incompressible fluid past a stationary semi-infinite flat plate was studied by Blasius [1], Howarth [2] and these have been discussed in books by Schlichting [3], Pai [4], Rosenhead [5]. The boundary layer adjacent to continuous moving solid surfaces was first investigated by Sakiadis [6]. In his pioneering papers the boundary layer differential equations for continuous moving flat and continuous moving cylindrical surfaces were presented. Flow of this type represents a new class of boundary layer problems with solutions substantially different from those boundary layer flow on stationary surfaces. However, the boundary conditions for these two different classes of problems are different. A polymer sheet or filament extruded continuously from a die, or a long thread travelling between a feed roll and a wind-up roll, is an example of a moving continuous surface. Sakiadis' paper also contained the development of the integral momentum equation for both flat and cylindrical continuous surfaces. In a subsequent publication Sakiadis [7] presented an exact and an approximate

solution for the case of laminar flow and flat geometry. An approximate solution thought to be valid for turbulent flow and flat geometry was also given. Later, Sakiadis [8] treated the cases of laminar and turbulent boundary layers on a continuous circular cylinder by an integral method. Koldenhof [9] working independently, used an integral technique somewhat different from that used by Sakiadis to treat the problem of a laminar boundary layer on a continuously moving cylinder. The equations of Koldenhof are essentially the same as Sakiadis. Koldenhof also presented the results of an experimental investigation of drag on continuous cylinders and good agreement was obtained between the theoretical and experimental results. Griffith [10] presented experimental data for shear stress and velocity profiles in boundary layer adjacent to continuous moving cylindrical surfaces. These data qualitatively agreed with the data of Koldenhof. Griffith also reasoned that for the boundary layer associated with polymer spinning, the boundary layer equation of energy could be linearized with loss of accuracy. Griffith presented solutions to the linearized equation of energy in his paper. The heat transfer aspect of Sakiadis' problem was studied by Tsou, Sparrow and Goldstein [11] and in case of continuously moving cylinders, it was discussed by Pechoc [12], Bourne and Elliston [13], Rotte and Beck [14], Gampert [15, 16] and Karmis and Pechoc [17]. In Ref. [11], the viscous dissipation effects were not considered. So on taking into account the viscous dissipative effects, the temperature field was studied by Soundalgekar and Murty [18] and similarity solutions were presented. In Ref. [11], the plate temperature was assumed as constant. Hence, Soundalgekar and Murty [19] studied the effects of variable temperature of the plate on the heat transfer in flow past a continuously moving plate. However, in these papers [1, 19] the plate was assumed to be impermeable and hence it was not possible to apply suction/injection. The significance of suction/injection for the boundary layer control has been well recognised. It is often necessary to prevent (or postpone) separation of the boundary layer to reduce drag and attain high lift values. The effect of

suction is to shift the point of separation in the direction of motion or else the separation is completely avoided. With this aim in mind, Soundalgekar and Murty [20] studied the effect of suction/injection on flow past a continuously moving flat plate with variable temperature. In Ref. [20], another physical situation which is of interest is the study of heat transfer flow past a continuously moving flat plate in the presence of suction and injection with heat flux boundary condition. Such a study has not received the attention of any research worker and hence it is proposed to study in this paper.

In Sec. 2, the mathematical analysis has been presented and in Sec. 4, the conclusions have been set out.

2 Mathematical analysis

Here x -axis is taken along the sheet in the direction of motion of the sheet, and y -axis is taken normal to it. The sheet is assumed to be moving with a uniform velocity U_0 . If u and v are the velocity components along x and y -axis respectively, then the boundary layer equations governing the flow and heat transfer are given by the following equations:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{2}$$

Here, T is the temperature of the fluid, ν the kinematic viscosity, $\alpha = (k/\rho c_p)$ the thermal diffusivity, c_p is the specific heat at constant pressure, ρ is the density and k is the thermal conductivity of the fluid.

The boundary conditions are

$$u = U_0, \quad v = V_0(x), \quad \frac{\partial T}{\partial y} = -\frac{q_w}{k}$$

where q_w is the heat flux per unit area.

$$u = 0, \quad v = 0, \quad T = T_\infty \text{ as } y \rightarrow \infty. \tag{3}$$

Introducing the following similarity transformations

$$\eta = y \sqrt{\frac{U_0}{\nu x}}, \quad u = U_0 f', \quad v = \frac{1}{2} \sqrt{\frac{U_0 \nu}{x}} (\eta f' - f) \tag{4}$$

$$\phi = \frac{k(T - T_\infty)}{q_w} \sqrt{\frac{U_0}{\nu x}}$$

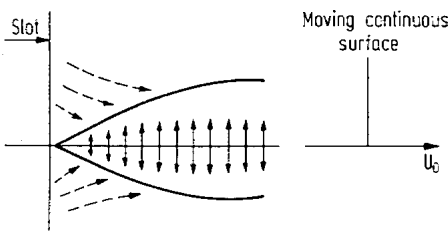


Fig. 1. Boundary layer on a moving continuous flat surface with injection

$$Pr = \frac{\mu c_p}{k} \text{ (Prandtl Number)}$$

in Eqs. (1)–(3), we get

$$f''' + \frac{1}{2} f f'' = 0 \tag{5}$$

$$\phi'' + \frac{Pr}{2} (f \phi' - f' \phi) = 0 \tag{6}$$

and the boundary conditions are:

$$f(0) = -\frac{2 V_0(x)}{U_0} \sqrt{Re} = f_w, \quad f'(0) = 1, \quad \phi'(0) = -1.$$

$$f'(\infty) = 0, \quad \phi(\infty) = 0. \tag{7}$$

Here $Re = \frac{U_0 x}{\nu}$ is the Reynolds Number.

In order to have a physical point of view of the problem, numerical calculations of Eqs. (5) and (6) subject to boundary conditions (7) are carried out for Prandtl number $Pr = 0.7$ (air) and the parameter f_w which represents the velocity of suction or injection when $f_w > 0$ and $f_w < 0$ respectively. The temperature profiles are shown in Fig. 2. Numerical values of $\phi(\eta)$ and $\phi'(\eta)$ have been calculated and they are entered in Tables 1 and 2. These results will be found useful in chemical industry and fibre industry.

3 Method of solution

The Eqs. (1) and (2) are to be solved the corresponding boundary conditions (3). The velocity boundary layer equation is non-linear in character and is independent of the temperature field and therefore can be solved independently to the thermal boundary layer equation, whereas the thermal boundary layer equation depends on the

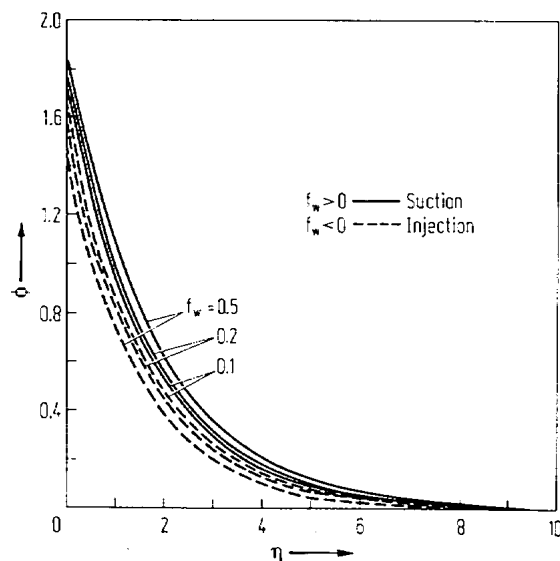


Fig. 2. Temperature profiles, $Pr = 0.7$ (air)

Table 2. (continued)

η	Values of $-\varphi'(\eta)$ $Pr = 0.7$ (air)					
	f_∞					
	0.5	0.2	0.1	-0.1	-0.2	-0.5
8.8	0.00951	0.00729	0.00659	0.00529	0.00470	0.00315
9.0	0.00858	0.00654	0.00590	0.00471	0.00417	0.00277
9.2	0.00774	0.00586	0.00528	0.00420	0.00371	0.00244
9.4	0.00699	0.00526	0.00473	0.00374	0.00329	0.00214
9.6	0.00631	0.00472	0.00423	0.00333	0.00292	0.00189
9.8	0.00569	0.00424	0.00379	0.00297	0.00260	0.00166
10.0	0.00514	0.00380	0.00339	0.00264	0.00230	0.00146
10.2	0.00464	0.00341	0.00304	0.00235	0.00205	0.00128
10.4	0.00419	0.00306	0.00272	0.00209	0.00182	0.00113
10.6	0.00378	0.00274	0.00243	0.00187	0.00161	0.00099
10.8	0.00341	0.00246	0.00218	0.00166	0.00143	0.00087
11.0	0.00308	0.00221	0.00195	0.00148	0.00127	0.00077
11.2	0.00278	0.00198	0.00174	0.00132	0.00113	0.00068
11.4	0.00251	0.00178	0.00156	0.00117	0.00100	0.00059
11.6	0.00226	0.00159	0.00140	0.00104	0.00089	0.00052
11.8	0.00204	0.00143	0.00125	0.00093	0.00079	0.00046
12.0	0.00184	0.00128	0.00112	0.00083	0.00070	0.00040
12.2	0.00166	0.00115	0.00100	0.00074	0.00062	0.00035
12.4	0.00150	0.00103	0.00090	0.00066	0.00055	0.00031

velocity field and can be solved only when the velocity field is known. The first equation is of the third order and the latter is of the second order. For the functions determined by the Eqs. (1) and (2), three boundary conditions at $\eta = 0$ and two at $\eta = \infty$. We have solved the Eqs. (1) and (2) numerically using Taylor series method on CDC-3600 computer with the interval size $\Delta\eta = 0.05$. To satisfy the boundary conditions (1-2), three of which at $\eta = 0$ and the remaining two are given at $\eta = \infty$, we compute the solutions of (1-2) assuming guess values of $f''(0)$ and $\varphi(0)$. The solutions may not in general satisfy the boundary conditions $f'(\infty) = 0$ and $\varphi(\infty) = 0$. Depending on whether the computed value of $f'(\infty)$ is less than zero or greater than zero, one either increases or decreases the guess value assumed for $f''(0)$. This process is continued until $f'(\infty)$ tends to zero asymptotically. In a similar manner $\varphi(\infty)$ is also corrected. This is the well known "Shooting Method" of solving a two point boundary value problem.

4 Conclusions

1. An increase in suction value resulted in the increase in temperature.
2. An increase in injection value resulted in the decrease in temperature.
3. An increase in suction value leads to an increase in $\{-\varphi'(\eta)\}$ the rate of heat transfer.
4. An increase in injection value leads to a fall in rate of heat transfer, $\{-\varphi'(\eta)\}$.

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