# STABILITY PARAMETERS AND THEIR INTER-RELATIONSHIPS AT THE NAVIFACE

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#### ABSTRACT

Different forms of stability parameters used for the estimation of fluxes and studies on the structure of surface boundary layer of the marine atmosphere and their inter-relationships under the varying conditions of thermal stratifications at the naviface are reviewed The conversion of one stability parameter into the other, with the aid of the stability functions, is also considered.

## INTRODUCTION

Estimations of the fluxes of heat and momentum in the surface boundary layer (sbl) (Fig. 1A) over the sea, under varying stability conditions, is of prime concern for investigations in air-sea interaction problems. These estimates will provide the basic information to investigate the small and large-scale processes like turbulence structures within the sbl in consequence to the structure of the marine atmosphere, deviations of sea surface temperature and the resulting anomalies of oceanic and atmospheric circulations, etc. Most of the direct and indirect methods for the estimations of fluxes of momentum and heat through the sbl of the marine atmosphere have been reviewed by Anto and Hasse (1980) and Busch (1977) and discussed as in Favre and Hasslemann (1978). Comprehensive reviews on the investigations relating to the structure of turbulence, exchange of fluxes, etc. were made by Haugen (1973), Businger (1975) and Busch (1973).

A detailed account of the stability parameters and their inter-relationships for the estimations of the fluxes of momentum and heat at the naviface (air-sea inter-face) and the structure of the boundary layers of the marine atmosphere, may perhaps be of great utility for application to studies such as the convective instability—a characteristic feature of the monsoonal circulation. The data collected under the International Monex experiment could be subjected for a detailed analysis on the thermal stratifications of SBL with the aid of these stability parameters.

The present note attempts to review the different forms of commonly used stability parameters and their inter-relationships with stability functions. The discussions will be oriented to those parameters and functions under varying thermal stratifications in the SBL of the marine atmosphere.

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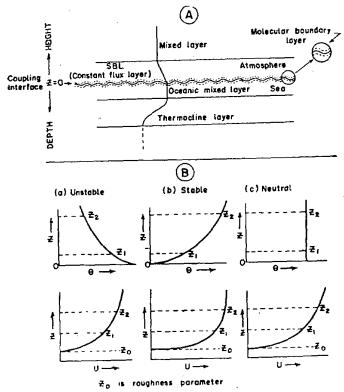


Fig. 1. (A) Surface loundary layer (SBL)-A Schematic representation;

(B) Characteristic profiles of temperature and wind within SBL.

## THEORETICAL ASPECTS

The transfer of fluxes through the SBL depends on the prevailing turbulent structure. The stratified nature of this SBL to great extent determines the nature of the transport processes. The effects of the thermally stratified layer control the energy inputs turbulence through buoyancy forces. Under stable stratificattons, the turbulence gets suppressed, while free convection results when unstability prevails. As the SBL is generally in a thermally stratified state, the variations in the fluxes are dependent on the stability parameters and reflect the prevailing turbulence in the SBL of the marine atmosphere-

Some of the theoretical considerations of these paramaters have been dealt by Lumley and Panoisky (1963). Roll (1965), Webb (1965), Kraus (1967 and 1972) and Businger (1973) in

connection with applications for various studies. The most commonly used forms of stability parameters, in relation to the transfer processes of heat and momentum at the naviface and the structure of the boundary layers of the overlying marine atmosphere are

$$\frac{Z}{L}, \frac{Z}{L'}, R_i, R_f \text{ and } R_{i_{Bulk}}$$
where 
$$\frac{Z}{L} = -\frac{g}{T} \cdot \frac{H_v/C_p \rho}{u_*^3} \cdot kZ;$$

$$\frac{Z}{L'} = \frac{g}{T} \cdot \frac{\partial \theta_v/\partial z}{u_* \partial u/\partial z} \cdot kZ;$$

$$R_i = \frac{g}{T} \cdot \frac{\partial \theta_v}{\partial z} / \left(\frac{\partial u}{\partial z}\right)^2 = \left(\frac{N}{\partial u/\partial z}\right)^2$$

$$R_i = -\frac{g}{T} \cdot \frac{H_v/C_p \rho}{u_*^2 \partial u/\partial z}$$

$$= \frac{g}{T} \cdot \frac{\overline{w'T'}}{\overline{u'w'} \partial u/\partial z}$$

N, the Brunt Väisälä frequency, is real in a stable and imaginary in an unstable one.

$$R_{\rm i~Bulk} = \frac{g}{T} \cdot \frac{(\bar{\theta}_{\rm v,1} - \tilde{\theta}_{\rm v,0})}{\bar{U}^2} z$$

where  $(\overline{\theta}_{v,1} - \overline{\theta}_{v,0})$  is the virtual air-sea temperature difference and  $\overline{U}$  the mean wind speed.

Since water vapour is less dense than air, the influence of humidity on the stability parameter has to be considered. This could be met by applying a virtual correction (terms with suffix  $\nu$ ) to the observed temperature at the desired level-

$$\frac{\partial \theta_{v}}{\partial Z} = \frac{\partial \theta}{\partial Z} + 0.6T \frac{\partial q}{\partial z}$$

$$\frac{H_{v}}{C_{p}\rho} = \frac{H}{C_{p}\rho} + 0.6T \frac{E}{\rho} \quad \text{where} \quad E = \rho \overline{q'w'}.$$

These stability parameters are used to investigate properties of the turbulence in the surface boundary layer under different conditions defining the stratification (Fig. 1B). Following Monin and Obukhov (1954), the stability functions (dimensionless gradients) or universal functions are defined by

$$\phi_{\rm m} = \frac{kZ}{u_*} \cdot \frac{\partial u}{\partial z}$$

$$\phi_{\rm H} = \frac{kZ}{\theta_*} \cdot \frac{\partial \theta}{\partial z} \text{ and }$$

$$\theta_{\rm E} = \frac{kZ}{\theta_*} \cdot \frac{\partial q}{\partial z}$$

Stability functions recommended for use at sea are discussed in Anto and Hasse (1980) and Hasse, Gruenewald, Wucknitz, Dunckel and Schriever (1978).

#### INTER-RELATIONSHIPS

From given stability functions for momentum and heat  $(\phi_m \text{ and } \phi_H)$ , in terms of one stability parameter, it is possible to convert the different parameters into each other. This is done by the following set of equations

$$\phi_{\text{m}} = \frac{kZ}{u_{*}} \cdot \frac{\partial u}{\partial z} \text{ where } u_{*} = \sqrt{T/\rho}$$

$$\phi_{\text{H}} = \frac{kZ}{\theta_{*}} \cdot \frac{\partial \theta_{\text{v}}}{\partial z} \text{ where } \theta_{*} = -\frac{H}{C_{\text{p}}\rho u_{*}}$$

$$\frac{Z}{L} = -\frac{g}{T} \cdot \frac{H_{\text{v}}/C_{\text{p}}\rho}{u_{*}^{3}} kZ;$$

$$\frac{Z}{L'} = \frac{g}{T} \cdot \frac{\partial \theta_{\text{v}}/\partial z}{u_{*}\partial u/\partial z} kZ;$$

$$R_{\rm I} = \frac{g}{T} \cdot \frac{\mathcal{D}\theta_{\rm v}}{\mathcal{D}z} / \left(\frac{\mathcal{D}u}{\mathcal{D}z}\right)^2 \text{ and}$$

$$R_{\rm I} = -\frac{g}{T} \cdot \frac{H_{\rm v}/C_{\rm p}\rho}{u_{\rm w}^2 \mathcal{D}u/\mathcal{D}z}$$

The inter-relationships between the above stability parameters and stability functions are

$$R_{\rm i} = \frac{\phi_{\rm H}}{\phi_{\rm m}} \cdot R_{\rm f} = \frac{\phi_{\rm H}}{\phi_{\rm m}^2} \cdot \frac{Z}{L} = \frac{J}{\phi_{\rm m}} \cdot \frac{Z}{L'} \qquad \qquad . \tag{A}$$

$$R_{\rm f} = \frac{\phi_{\rm m}}{\phi_{\rm H}} \cdot R_{\rm i} = \frac{I}{\phi_{\rm m}} \cdot \frac{Z}{L} = \frac{I}{\phi_{\rm H}} \cdot \frac{Z}{L'} \qquad .. \qquad .. \qquad .. \qquad (B)$$

$$R_{\rm f} \cdot \phi_{\rm m} = \frac{\phi_{\rm m}^2}{\phi_{\rm H}} \cdot R_{\rm i} = \frac{Z}{L} = \frac{\phi_{\rm m}}{\phi_{\rm H}} \cdot \frac{Z}{L'} \qquad \qquad .. \qquad .. \qquad (C)$$

$$\phi_{\rm m} \cdot R_{\rm i} = \phi_{\rm H} \cdot R_{\rm f} = \frac{\phi_{\rm H}}{\phi_{\rm m}} \cdot \frac{Z}{L} = \frac{Z}{L'} \qquad \qquad .. \qquad .. \qquad (D)$$

From the above inter-relationships in A, B, C, and D, it is seen that Z/L and other stability parameters are directly proportional to Z. On any given occasion, the characteristic height represented by Monin-Obukhov length L is a constant and independent of height. Under these conditions, the stability parameter Z/L is proportional to  $Z_1$ . The other stability parameters depend mainly on the gradients of wind and temperature and thereby proportional to  $Z_2$ . For example, at greater heights, the gradients deviate from the neutral value. This deviation would be small in unstable and large for stable conditions (Webb, 1965).

To obtain the relations between R<sub>i Bulk</sub> and the other stability parameters, an iniegration of the stability function, as introduced by Panofsky (1963), is used-

$$R_{i \text{ Bulk}} = \frac{g}{T} \cdot \frac{(\theta_{v,1} - \theta_{v,0})}{U^2} \cdot Z$$

 $R_{i Bulk}$  is related to Z/L (where Z is the total height of the layer under considration) by

$$\frac{Z}{L} = \frac{\left[\ln(Z_1/Z_{\circ,u}) - \psi_{\mathfrak{m}}\right]^2}{\left[\ln(Z_2/Z_{\circ},\theta) - \psi_{\mathfrak{H}}\right]} \cdot R_{i \text{ Bulk}}$$

where  $\psi$  is the integrated stability function expressed as

$$\psi = \int_{\zeta_0}^{\zeta_1} \frac{1 - \phi(\zeta)}{\zeta} \cdot d\zeta$$

$$= e g \int_{Z_0/L}^{Z_1/L} \frac{[1 - \phi(Z/L)]}{Z/L} \cdot d(Z/L)$$

and Z<sub>1</sub> and Z<sub>2</sub> are the different levels of measurement (Fig. 1B).

The stability function ( $\phi$ ) actually depends on which stability parameter is selected. Usually,  $\phi$  is expressed in terms of Z/L. Dyer (1974) has discussed about the suitable stability parameter Z/L. Further, Dyer has thoroughly reviewed the stability functions and commented on the suitable stability functions for different stability conditions. A conversion from one stability parameter to the other may seem uncertain, since the stability functions are not too well known. In practice, this uncertainty is unimportant.

#### DISCUSSION

The different stability parameters described above have originally been defined for different reasons. These parameters primarily reflect the turbulent transfer processes in the SBL. Monin (1970) and Panofsky (1974) have qualitatively reviewed some aspects of boundary layers.

For large positive values of Z/L and similarly the other stability parameters, the turbulence gets suppressed and might even be destroyed during stable temperature stratification. During small values of Z/L, the flow remains near neutral in the surface layer where the buoyancy effects are insignificant. Large negative values of Z/L indicate the free convection regime where the convective turbulence dominates the machanical generation. The identification of the free convection regime will enable to use the free convection formulae for the reliable estimation of heat flux and friction velocity from the temperature and wind profiles (Sundararajam and Macklin, 1976). Small negative value of Z/L gives indications of shear convection effects. Under unstable conditions, the values of different stability parameters are nearly the same.

 $R_f$  was defined to give the ratio of the buoyant to mechanical production of turbulence. Since this involves knowledge of the fluxes of heat and momentum, which are often not known,  $R_f$  has been approximated by  $R_i$ , which is determined from the profiles of temperature and wind as mentioned above and thus  $R_f$  and  $R_i$  are height dependent. This is a drawback from a more theoretical point of view, where it is assumed that the fluxes at the interface are the relevant parameter of the problem for the surface layer. This led to the definition of the Monin-Obukhov length L, which is determined solely in terms of fluxes at the interface. Thus Z/L is often used as a stability parameter in theoretical derivations. Again, since turbulent fluxes are often not available from direct measurements, Z/L is approximated by Z/L'. For practical purposes, this approximation implies that heat flux is required for the determination of Z/L and for Z/L' the profiles of wind and temperature are required.

As has been said above, while R<sub>f</sub> and Z/L have a simple physical meaning, they are difficult to determine because measurements of turbulent fluxes are usually not available. Although profile measurements are somewhat less demanding in experimental effort, in the overwhelming majority of cases in applications only mean meteorological variables are available. This has led to the introduction of the bulk Richardson number, R<sub>i</sub> Bulk. The bulk Richardson number is defined in terms of mean wind speed and air-sea temperature difference and thus is suitable for use with routine marine meteorological data. Beyond that Hasse, Gruenewald, Wucknitz, Dunckel and Schriever (1978) used a bulk Richardson number for determination of

stability parameters, although profiles were available. The reason for this is that the bulk Richardson number is less sensitive to experimental errors compared to the gradient Richardson number. As indicated above, both Z/L and Richardson number are functions of height. Hence these stability parameters do not easily characterise a given situation. Therefore, Resch and Selva (1978) have preferred  $R_{i,Bulk}$  to study the different stability conditions on turbulent transfer processes. The air-sea temperature difference and 10 m level wind speed entering in the former one is about one order of magnitude larger than the vertical differences within the height range used for profile measurement in case of the later one.

From the aforementioned discussions, Z/L and  $R_i$  give easy access for practical applications like investigations on the structure of the boundary layer, estimations of the fluxes of momentum and heat etc. For studies pertaining to the monsoonal circulations, Z/L would be an ideal stability parameter. Convective instability being a feature of the monsoonal circulation, the large negative values of Z/L would give an image of the convective turbulence. For large scale maritime meteorological and oceanographic applications, the bulk Richardson number is a handy tool, and it is worthwhile to note that Z/L and other stability parameters may be derived from it.

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### LIST OF SYMBOLS

Ź		Height from sea surface	$q_{*}$		Scaling humidity
$Z_{\circ}$	· —	Roughness parameter	ζ		Dimensionless height
L		Monin - Obukhov length	N	_	Brunt Väisälä frequency
Z/L	-	Stability index	$\theta_{ m v}$		Virtual potential
Z/L'		An approximation to $Z/L$			temperature
$R_{\mathrm{i}}$		Gradient Richardson	$\boldsymbol{g}$	_	Acceleration due to gravity
		number	T		Air temperature
$R_{ m f}$		Flux Richardson number	$T_{ m v}$		Air temperature after
R <sub>i</sub> Bulk		Bulk Richardson number			applying correction for
T		Momentum flux			moisture
H		Sensible heat flux	p		Air density
E		Water vapour flux	$C_{\mathrm{p}}$		Specific heat
u,v,w		Wind components	k		von Karman constant
		(' indicates instantaneous	ψ		Integrated stability function
		fluctuations)	$\phi_{m}, \phi_{H}, \phi_{E}$	_	Stability functions for
$u_*$		Frictional velocity			momentum, heat and water
$g_*$	_	Scaling temperature			vapour respectively.

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