

## Phase Spectral Composition of Wind Generated Ocean Surface Waves

M J VARKEY

National Institute of Oceanography, Dona Paula, Goa 403 004, India

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A study of the composition of the phase spectra of wind generated ocean surface waves is carried out using wave records collected employing a ship borne wave recorder. It is found that the raw phase spectral estimates could be fitted by the Uniform probability distribution in all cases with widely varying confidence levels. Gaussian fits are found suitable with varying confidence levels for smoothed phase spectra for narrow and wide band variance spectra except when the spectra are well differentiated into 2 wave systems with a dominating local sea. For this type of sea states, a directional tuning in the directional phase spectra could be expected. A new spectral width parameter which is equally applicable to spectra of narrow band, wide band and multip peaked nature is also defined.

Wind generated ocean surface wave systems are considered as random phenomena. But the process is actually non-random when higher orders are considered. The general component view of the ocean surface as a superimposition of a number of wavelets of different frequencies and amplitudes whose phases are distributed randomly is correct only when non-linearities are strictly absent. The assumption of randomness is roughly valid in a well dispersed swell system only. Normal wind-generated waves are generally considered to be weakly non-linear. But wind waves are strongly non-linear when they are in the growing phase in the generating area and in the dissipating phase in shallow waters and breaking sea states.

Waves of different frequencies are also known to interact non-linearly, transferring the component energies between themselves or to new excited (bound) components. These interactions are considered to be influenced by the non-linear nature of the individual components and the phase relationships that exist in between them. The non-linear interaction<sup>1-4</sup> due to phase coupling would be strong and more effective in the energy containing region (spectral peak) and between its higher harmonics due to the closeness of H/L ratios in the spectral peak and the suitable condition of prolonged 'locked' interaction between the harmonics and at the spectral peak.

What happens in the 'locked' phase relationship is not known. It can be suggested that the particle movements and the resulting orderly turbulence (in well dispersed systems only) would have a primary role in the non-linear transfer. In this paper an attempt is made to study the phase spectrum of wind

generated ocean surface waves recorded using a ship-borne wave recorder for any type of deviations or abnormalities which might exist due to non-linearities in the wave profile and which might go unnoticed in the usual Uniform fits. Attempt is also made to trace and propose any changes of the phase spectrum resulting from interaction with other wave systems and the effect of this factor on the sea state.

A word of caution at this juncture would be necessary. As, a sampled wave profile (presently at 1 s interval) would lose many of its finer features differently for different frequency ranges, a number of wave records with very different spectra are selected to arrive at some relative results due to the interaction of different wave systems.

### Materials and Methods

Wave data were recorded using a ship-borne wave recorder on board *R V Gaveshani*. The sensors were fixed at a depth of 2.1 m below the mean water level. Data were recorded<sup>5</sup> for 15 min durations at 2 stations whose depths were 250 and 500 m off the east coast of India at about 16° 30' N from 24 September to 4 October 1980. These records were digitised at 1 s intervals and the resulting time series were used for the present study.

Even though wave records were available for 15 min durations, only stabilised portions of the records were digitised giving always less than 900 digitised values at 1 s intervals. The data series thus obtained were analysed for their variance and phase spectra using Fast Fourier Transform (FFT) techniques<sup>6</sup>. These programs would work only with specific number (N) of samples equal to  $2m(m > 0)$ . In the present case only 512 ( $= 2^9$ ) values could be

used due to this limitation. Even though N could be adjusted to 1024 ( $= 2^{10}$ ) by adding zeros, this method was not adopted since it requires variance adjustments after the computations and since  $N = 512$  was found suitable for the present study. The spectral densities (after removing noise) were corrected for the attenuation effect<sup>7</sup> due to the submersion of the sensors by 2.1 m. The attenuation corrected raw variance spectra and the phase spectra were smoothed by a running average method with 11 points resulting in about 22 degrees of freedom.

**Results and Discussion**

Fig. 1 shows the different variance spectra selected for the present study for reasons mentioned in the Introduction. Each spectrum was observed at a particular time in a continuous wave evolution sequence for a period of 9 d and corresponds to different local wind fields, sea systems and swell systems. Even though the spectra covered a wide range, the significant wave heights ( $H_s = 4\sqrt{m_0}$ ) ranged only from 1.39 m (WAPD 16) to 0.56 m (WAPD 11) showing that the sea states were comparatively mild. Some of the relevant spectral statistics which are used in the following discussion are given in Table 1.

The raw phase spectral values were grouped into classes (width = 0.2618) about the mean. Uniform probability law was applied to the grouped raw phase estimates within the valid sensor ranges (0.05 to 0.33 Hz). The distributions (Fig.2) are reasonably good fits (Table 1) with varying confidence levels justifying the application of Uniform probability law. Fits for the records WAPD01, WAPD08, WAPD14,

WAPD16 and WAPD19 are found to be acceptable only within 2 and 5% levels. No reason could be found for these low level confidence levels except for the reason that as the spectral (variance) constitution of the records change, variations in the structure of phase spectrum could be expected (the basic assumption of strict random phase structure is valid only for a narrow band spectrum). Another noteworthy feature is that some phase couplings, if present, between different Fourier components would not be detected in this analysis (see discussion on Fig. 6, also).

The smoothed phase spectral estimates were grouped into classes of width 0.2618 ( $15^\circ$ ) for fitting Normal probability law. This analysis was done to bring out any abnormalities in the phase spectra of various records which are not detectable by fits of the Uniform law. From Table 1, it is observed that for all the records Normal distribution is suitable with different confidence levels except for the records WAPD08 and WAPD07. From Fig. 1 it could be observed that WAPD08 represents a sea state wherein the locally generated sea variance ( $= 0.0532 \text{ m}^2$ ) is almost double the swell variance ( $= 0.0296 \text{ m}^2$ ). For WAPD07 the local sea variance is 4 times that of the swell variance (Table 1). This brings out the conspicuous abnormal behaviour of the smoothed phase spectra of sea dominated sea states.

Figs 1 and 3 and Table 1 suggest some vague effect of spectral band width on the goodness of fit described by the  $\chi^2$  values. But the conventional spectral width parameter  $\epsilon [(1 - m_2/m_0 m_4)^{0.5}]$  does not give a representative value unless normalised

Table 1—Different Statistics for Selected Records

[M = mean,  $\sigma$  = variance,  $\alpha_1$  = skewness,  $\alpha_2$  = kurtosis, P = approx. variance for the highest spectral peak, O = approx. variance for all other peaks]

Record	Amplit. distribu. (Normal)					Spect. statistics			Variance		Distri. smooth. Ph. Sp. (Normal)					Dist. raw Phas. Sp. Uni (ok)
	M	$\sigma$	$\alpha_1$	$\alpha_2$	ok (%)	Hs	$\epsilon$	$\gamma$	P(m <sup>2</sup> )	O(m <sup>2</sup> )	M	$\sigma$	$\alpha_1$	$\alpha_2$	ok (%)	
WAPD01	0	0.0445	-0.1411	3.1613	2	0.691	0.871	0.089	0.018*	0.008	3.022	0.5280	+0.054	2.638	20	2
WAPD03	0	0.0727	0.0817	3.0516	2	0.982	0.801	0.108	0.033*	0.021	3.377	0.5470	+0.038	2.461	10	10
WAPD14	0	0.0688	-0.1892	2.9966	2	1.025	0.760	0.191	0.038*	0.018	3.242	0.5634	-0.196	2.509	2	5
WAPD04	0	0.0610	-0.0839	3.2172	5	0.967	0.757	0.216	0.032*	0.025	2.816	0.4548	+0.013	2.023	15	25
WAPD08	0	0.0590	0.1081	2.7489	5	1.204	0.586	0.432	0.053*	0.030	2.876	0.7112	+1.098	3.962	N	5
WAPD16	0	0.0873	-0.0402	2.6580	N	1.391	0.652	0.372	0.021*	0.092	3.382	0.5715	-0.137	2.376	95	2
WAPD17	0	0.0699	-0.2722	3.3676	N	1.212	0.645	0.444	0.018*	0.075	3.045	0.5360	+0.304	2.612	30	75
WAPD19	0	0.0504	0.3069	3.0766	N	0.937	0.727	0.255	0.022*	0.016	2.940	0.6268	+0.324	2.296	10	5
WAPD10	0	0.0519	-0.0798	2.9575	5	0.912	0.739	0.232	0.023*	0.032	2.991	0.5121	+0.119	2.427	15	70
WAPD11	0	0.0249	0.0201	2.8375	5	0.557	0.842	0.238	0.011*	0.006	3.433	0.5803	-0.189	2.554	30	40
WAPD07	0	0.0428	0.0519	2.8693	5	1.229	0.490	0.293	0.064*	0.015	3.405	0.5794	+0.128	2.023	N	50

\*highest peak in swell region; † highest peak in sea region; N = No

VARKEY: PHASE SPECTRAL COMPOSITION

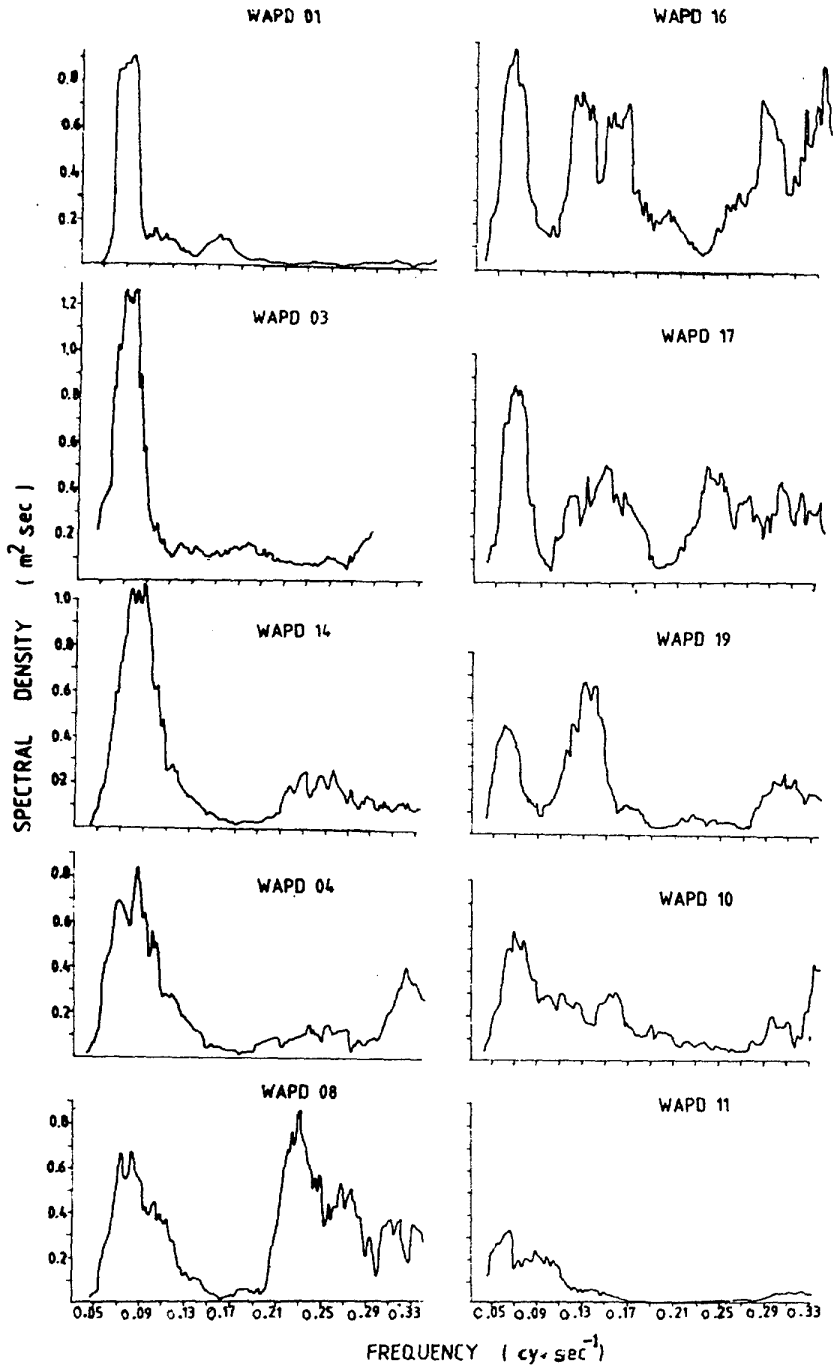


Fig. 1—Different wave variance spectra selected for the study

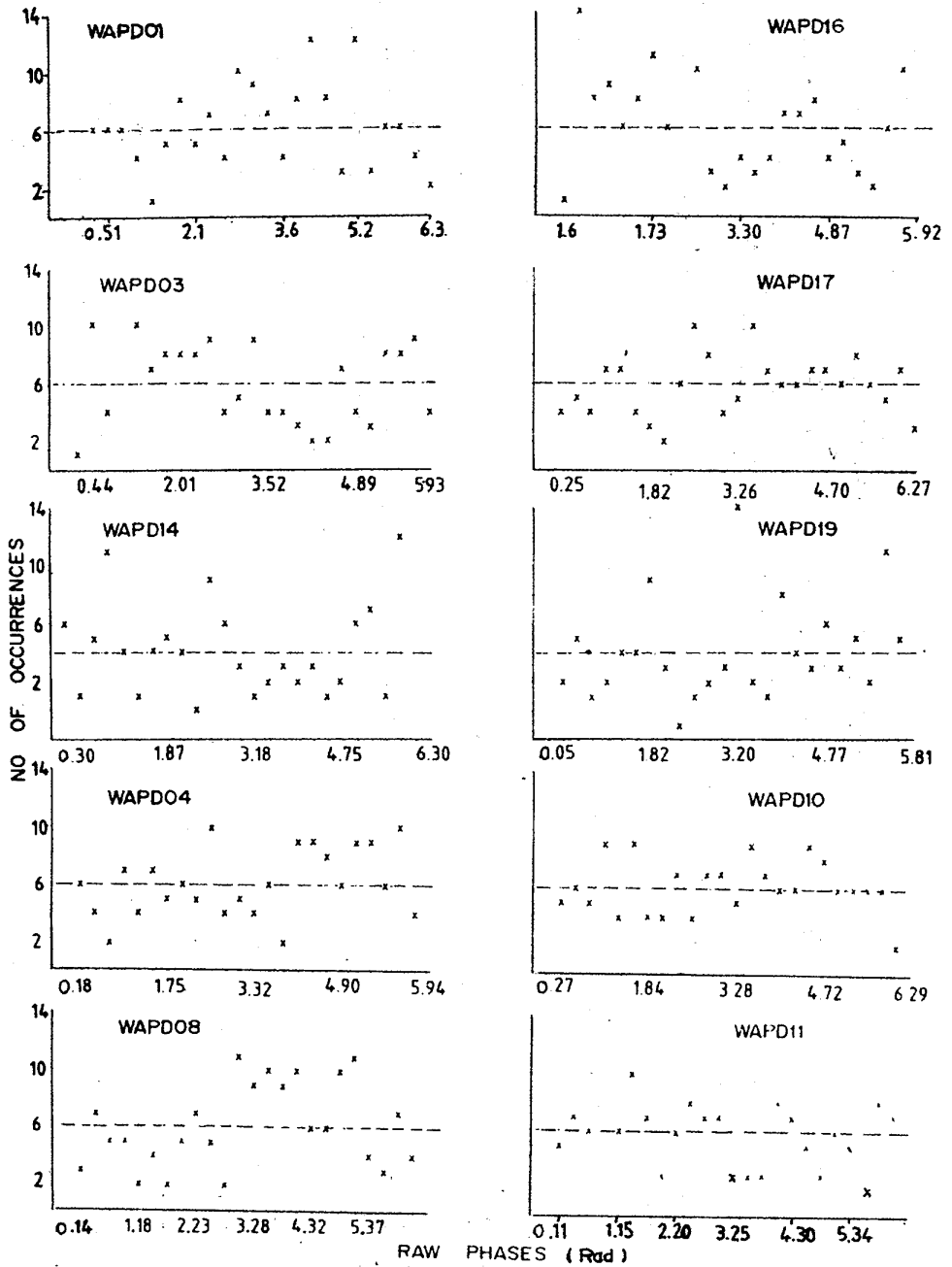


Fig. 2—Fits of Uniform probability law to raw phase spectral estimates [x = observed; --- = theoretical]

VARKEY: PHASE SPECTRAL COMPOSITION

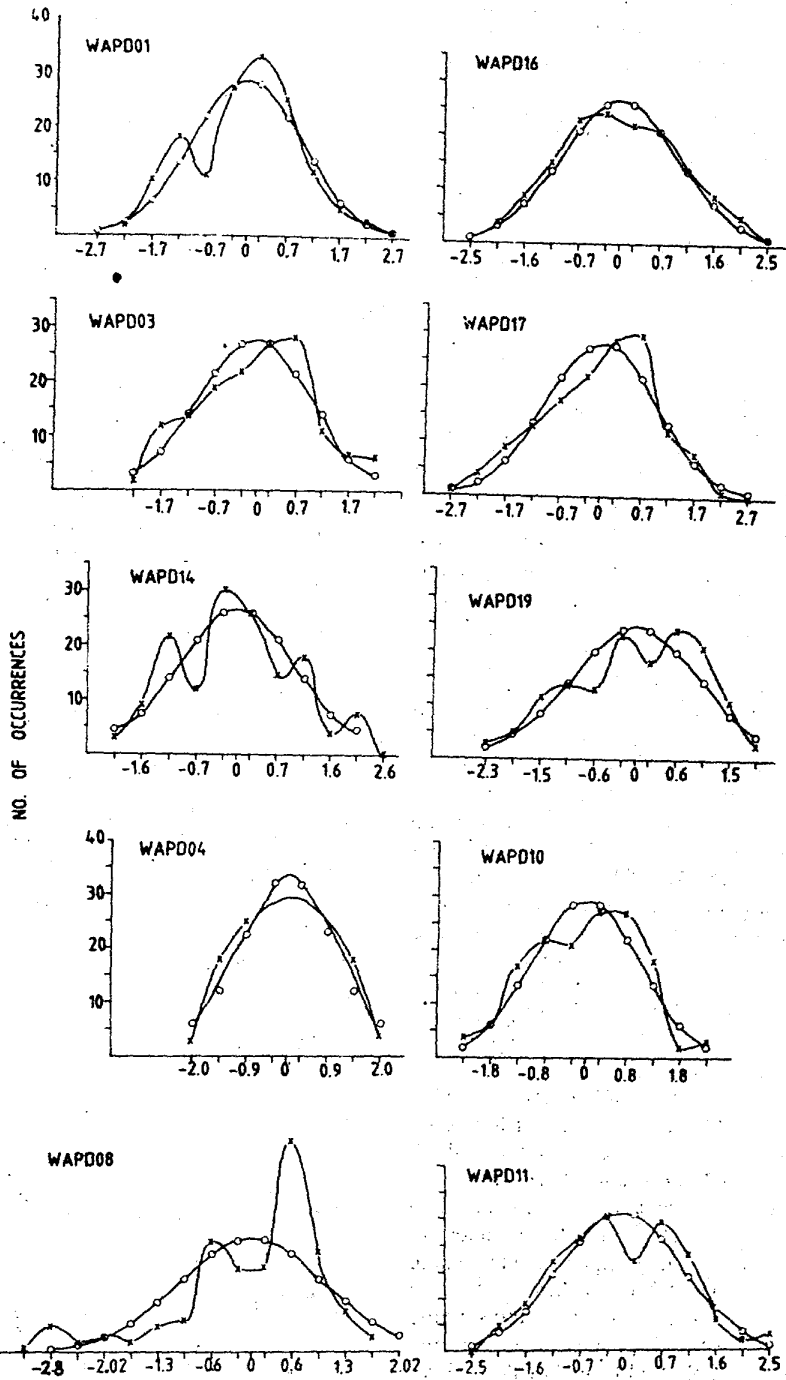


Fig. 3—Fits of Normal probability law to smoothed phase spectral estimates [x—x = observed; o—o = computed. X-axis is in standard Normal format]

spectral densities are used<sup>8</sup> and is applicable for narrow band single peaked spectra only. Hence it is clear that  $\epsilon$  is not the right parameter for the present study (Table 1).

What is required for the present purpose is an effective spectral width parameter which takes into account all the spectral peaks in any particular sea state and their relative dominance in relation to the maximum spectral peak. A wave system, here, is identified as one for which the half energy level is above the background spectral level. Such a parameter  $\gamma$  is defined below.

$$\gamma = (\gamma_p + \gamma_1(E_1/E_p) + \gamma_2(E_2/E_p) + \dots) / (f_u - f_l)$$
 where,  $\gamma_p$  = half energy band width of the highest spectral peak,  $\gamma_1, \gamma_2$  = half energy band widths of other spectral peaks,  $E_p$  = highest spectral peak value,  $E_1, E_2, \dots$  = other spectral peak values and  $f_u, f_l$  = highest and lowest frequencies. These values are estimated for different records (Table 1) and are plotted against the respective  $\chi^2$  values in Fig. 4. Another record very similar to WAPD08 with a large dominance of local sea (WAPD07) was also analysed and the results are presented in Table 1. It is clear from Fig. 4 that all records except WAPD07 and WAPD08 do not show any  $\gamma$  dependence of  $\chi^2$ . But the  $\chi^2$  values are large for these 2 records wherein the sea variances are very much larger than the swell variances (4 times for WAPD07 and twice for WAPD08). For 3 other records, WAPD16, WAPD17 and WAPD19, even though the  $\gamma$  values are large, the  $\chi^2$  values are found to agree with the general pattern. Similarly, if the 2 swell dominated records (WAPD01 and WAPD03) are considered, such large  $\chi^2$  variations are not noted. Hence it is clear that the large  $\chi^2$  values are due to the existence of a dominant wide sea spectral part and a narrow less peaked swell spectral part. Significant wave height ( $H_s$ ) shows a general linear trend (Fig. 4) of increase with  $\gamma$ , even though,  $H_s$  could be expected to show wide scatter with  $\epsilon$  or  $\gamma$ .

Gaussian fits (Fig. 5) are found generally suitable for initial wave amplitudes (uncorrected for depth attenuation) for all records (single or double peaked) except WAPD16, WAPD17 and WAPD19 (> 2 peaks and attenuation effect is very large at high frequencies). This is a normal result as the  $\gamma$  values are large for these records with multi-peaks. One notable feature is that for WAPD07 and WAPD08 Normal fits for the initial wave amplitudes are found suitable while Normal fits to smoothed phase spectra of the records are unacceptable (Table 1). For these 3 records (WAPD16, WAPD17 and WAPD19) Normal fits to smoothed phase spectra are found good as mentioned earlier (Table 1).

It can be observed from Fig. 6 that for WAPD01,

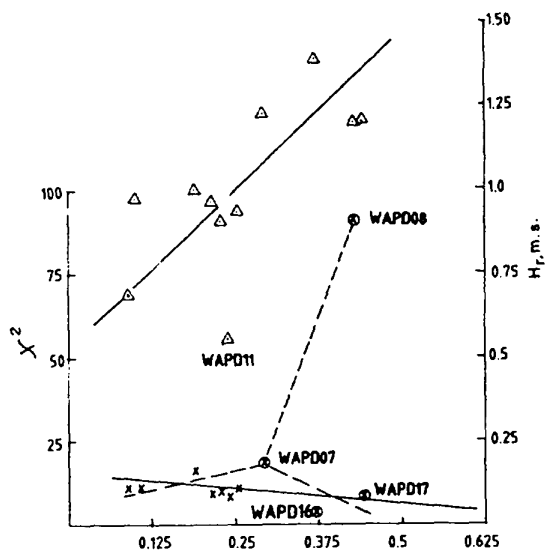


Fig. 4—Dependence of  $\chi^2$  [ $x$ — $\gamma$  and  $H_s$ ,  $\Delta$ — $H_s$ ] on effective spectral width

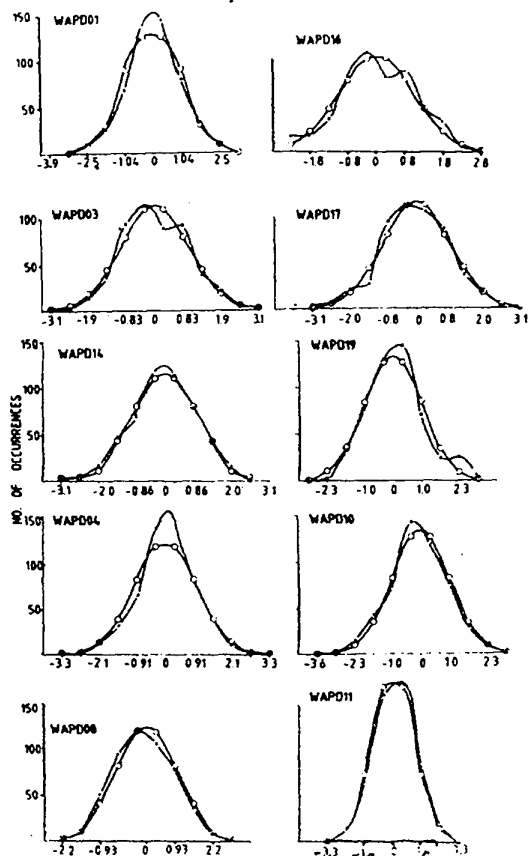


Fig. 5—Fits of Normal probability law to digitised wave amplitudes, ( $x$  — observed;  $o$  — computed. X-axis is in standard Normal format)

WAPD03 and WAPD11 the smoothed phase spectra look similar. From Fig. 1 it is seen that the variance spectral compositions are also similar with swell variances which are double those of the sea variances. In the case of WAPD08 the smoothed

phase spectrum shows consistency (frequency dependency) of 3.75 between 0.12 and 0.21 Hz (Fig. 6) wherein the spectral densities are very small. This consistency gives rise to the peak in the Normal fit in Fig. 3 at +0.55. Outside the region of consistency the smoothed phase spectrum shows wavyness similar to those observed in WAPD01, etc. WAPD07 (Fig. 6) also shows features similar to WAPD08 with consistency (linear trend) in the region 0.13 to 0.21 Hz with values ranging from 3.25 to 1.75. It is interesting that this type of frequency dependency in the smoothed phase spectra was not detectable from the raw phase spectra (Fig. 2). For multi-peaked spectra (WAPD16 and WAPD17) the smoothed phase spectra show consistencies and wavyness as a mixture of features from WAPD01, etc. and WAPD07, etc. From the above description it can be stated that the phase spectral parts of individual wave systems in a sea state have exclusive independent randomness and composite randomness (altogether) which sometimes get annulled as in the case of WAPD07 and WAPD08 wherein significant deviations are observed. This type of deviations are very likely to be a general feature for sea states wherein a well differentiated swell and sea interact each other with the sea playing a dominant role. As the sea state becomes multi-peaked ( $> 2$ ) the differentiation is lost and the composite random nature of the phase spectrum is restored.

From the above study it can be observed:

- (1) Gaussian fits are generally suitable for distributions of smoothed phase spectra.
- (2) The  $\chi^2$  values computed for the Normal fits to the smoothed phase spectra showed no dependence on an effective spectral width parameter for narrow band and wide band sea states except as at no. 4.
- (3) For sea states as described in no. 4 mutually exclusive and composite randomness in phase spectra of component wave systems could be suggested to be a normal feature.
- (4) If the peak spectral bands are well differentiated into 2 wave systems and if the local sea system dominates the swell system considerably, the phase spectral (smoothed) distributions show consistencies and the Normal law is not found suitable.
- (5) A similar feature as at no. 4 is not observed when the sea state is well differentiated into a dominant swell and a mild local sea.
- (6) A directional phase spectrum, if computed, would show some directional tuning for sea states described at no. 4.
- (7) While non-Normality of wave amplitudes point to non-linearities of the sea state, non-Normality of the smoothed phase spectrum need not necessarily suggest any non-linearity of the sea state.

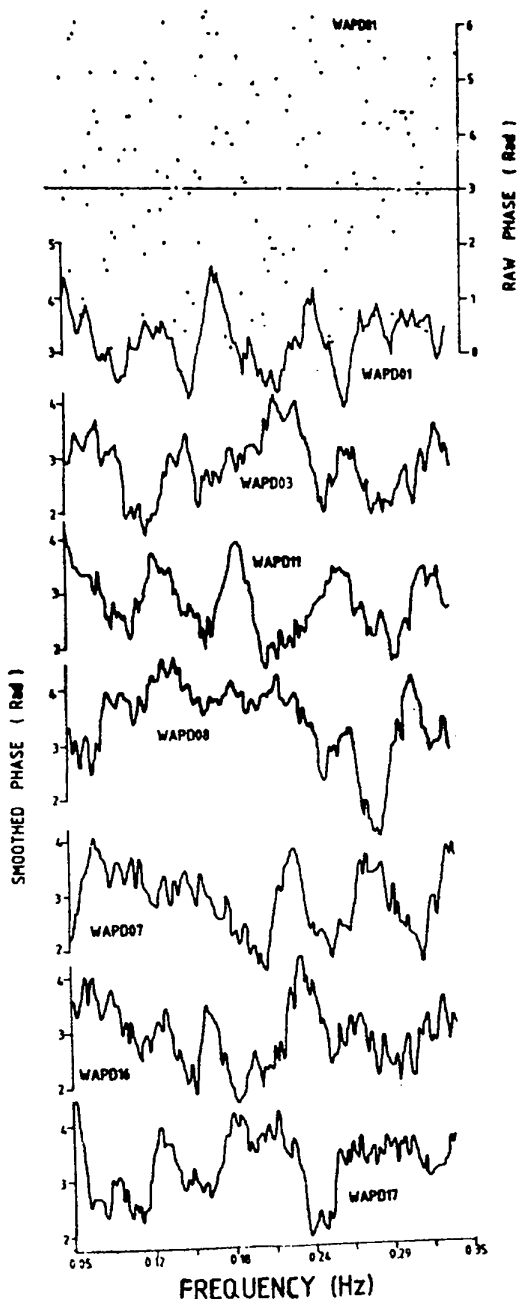


Fig. 6—Smoothed phase spectra for some selected records with one raw phase spectrum

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