

Numerical simulation of wind wave surface profiles with tuned phase spectra

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It is known that the phases of the individual harmonic components in a linear narrow band wave spectrum are uniformly random. It has been suggested by some workers that some sort of phase coupling and 'locking' between the different spectral components would lead to nonlinear surface waves. Conventional methods of wave recording and analysis inherently fail to record or to bring out the exact quantitative picture of nonlinear interactions due to recording and sampling limitations in relation to nonlinear waves with vertical and horizontal asymmetries especially in the case of breaking and shoaling waves. Hence, the inverse method of computing the surface profile from a known autospectrum using transformed (tuned) phase spectrum with coupling of different natures is tried and the results are presented here. Increased wave groupiness is found for higher spectral peak periods with phase coherency at the peak. Abnormally high breaking waves (freak waves) are generated when frequency dependent (by Normal probability law) phase spectrum is used. The results suggest that external factors like currents, wind, boundaries (bottom) and other waves systems can cause changes in the phase spectrum to give rise to nonlinear processes.

It is fairly well established that frequency dependent phase spectrum can give rise to nonlinear wave profiles¹⁻⁵. Lake and Yuen¹ proposed a model for a nonlinear wind wave system in which the sea system was completely characterised "by a single nonlinear wave train having a carrier frequency equal to that of the dominant frequency". The other spectral components were considered as non-dispersive bound wave components of the single dominant wave. The bound wave components were proposed to contain all the wave energy but other free waves of high frequency nature were modelled to exist along the surface of the dominant wave. They also concluded that to some sea states which are too linear, too multidirectional or too multisourced, their model might prove to be inapplicable. These results point to the fact that their model is pointedly applicable to the growing stage of the sea in which Mile's coupled shear flow mechanism⁶ with frequency selective feedback is dominantly active resulting in exponential growth. Elgar and Guza⁴ found that a nonlinear model for a sloping bottom predicts more accurately the evolution of energy spectra, coherence and other features. This model was explicitly developed for shoaling gravity waves. Hatori³ and Funke and Mansard⁷ found that the phase spectrum controls the nonlinear nature of the waves.

From the very fact that the reported¹⁻⁵ nonli-

near models predict more accurately the sea systems in growing and shoaling stages, it is clear that at these stages the sea states are nonlinear. It is known that the wave systems are largely nonlinear when they are at stages of growing and shoaling and are a result of interaction between multidirectional or multisourced (multip peaked) wave systems with a variant local wind and current directions. The spectral parameters and surface statistics of a narrow band swell system at an ideal recording site with no wind and currents are, exactly predictable by the linear wave model.

The above findings make it clear that nonrandomness in the phase spectrum leads to nonlinear sea states. Varkey⁸ tried to trace out trends or consistencies present in the phase spectrum of wind waves recorded using a ship borne wave recorder. He could bring out some anomalous structure in the smoothed phase spectra of two sea (local) dominated double peaked (with mild swell) autospectra. The above presented results revealed the possibility of the phase spectrum playing a very important role in nonlinear processes like wave growth, breaking and grouping and freak waves. Hence, an attempt is made here to generate some nonlinear wave profiles by different phase spectral tunings and the resulting surface profiles are studied for their distribution statistics like skewness, kurtosis and χ^2 values.

Methods

The surface wave profile as a time series (1 to N=512) is generated using Fast Fourier Transform (FFT) algorithms⁹ from a previously computed autospectrum with desired phase spectral tuning using the following inverse transform

$$x(n) = \left(\frac{1}{N}\right) \sum_{k=0}^{N-1} X(k) e^{i\phi(k) \sin(2\pi nk/N)}$$

for $n=0,1,\dots,(N-1)$... (1)

wherein, $X(k)$ is the complex valued transform for $k=0,1,\dots,(N/2)$. $X(k)$ is computed initially from a recorded surface wave profile sampled suitably to avoid aliasing. The phase spectrum $\phi(k)$ of the surface wave profile is given by

$$\phi(k) = \tan^{-1} \left[\frac{-B(k)}{A(k)} \right] \text{ for } k=0,1,\dots,(N-2) \dots (2)$$

where, $A(k) = R(k) \cos \phi(k)$

and $B(k) = R(k) \sin \phi(k)$... (3)

$\phi(k)$ is known to be randomly (Uniform) distributed for narrow band well dispersed swell systems.

In the introduction it was brought out¹⁻⁵ that frequency dependence of phase spectrum (or phase spectral tuning) would lead to nonlinear evolution of sea states. Hence, frequency dependence of different natures (ITUNE = 1,2,3,5 and 6) are introduced into Eq. (1) and the inverse transforms are worked out resulting in wave profiles with different distribution statistics. The variable ITUNE has the following meanings:

- ITUNE: 1 = the phase spectrum is Uniformly (computed pseudo-random sequence) distributed
- 2 = the phase spectrum is Normally (computed pseudo-random sequence) distributed
- 3 = the phase spectrum has phase coherence $\phi(k) = \phi(k+1) = \phi(k+2) \dots = \phi(k+n)$ at the spectral peak from $f(p)$ to $f(p) + (w/2)$ where $f(p)$ is the peak frequency and w the half energy width
- 5 = the original computed phase spectrum of the recorded profile
- 6 = smoothed phase spectrum from a running averaging (11 points) operation on the original phase spectrum of the recorded profile

A ship borne wave recorder (Institute of Oceanographic Sciences, UK) was used for recording

the sea surface⁸. Two recorded wave profiles (WAPD01 and WAPD07) were used for the computation of the wave profiles with different phase tunings; WAPD01 and WAPD07 correspond to a swell dominated sea state and a sea dominated sea state, respectively (Fig. 1). The energy content (represented by variance) of the dominant spectral peaks is approximately 60% for WAPD01 (0.05 to 0.12 Hz) and 80% for WAPD07 (0.2 to 0.33 Hz) of the total energy spread along the full valid sensor range (0.05 to 0.33 Hz) of the instrument. Different band pass filters are applied to tune the autospectra and the band widths are marked in Fig. 1. Spectral density values outside the band pass filtered frequency ranges are assigned insignificant values (0.00001) for computation purposes. Totally, 3 types of band pass filtered autospectra are subjected to different phase spectral tunings in addition to the full range (0.05 to 0.33 Hz) autospectra and the resulting surface profiles for 512 sec (N=512) are processed for their distribution statistics. The highest crests and the lowest troughs are marked along each profile for comparative purposes, since, different scales are used for plotting the profiles in the same figure.

Results and Discussion

Effect on narrow band swell—The computed wave profiles using a narrow band (0.05 to 0.12 Hz) swell (Fig. 1) obtained after removing the high frequency tail (variance=40%) from 0.12 to 0.33 Hz are shown in Fig. 2. Profile 1 (Fig. 2) is the same as recorded by the instrument including all wave components from 0.05 to 0.33 Hz (Table 1); the component waves are selectively attenuated according to an exponentially increasing function of frequency. From previous analyses of the same records it is found that high frequency components with periods < 3 sec are practically absent in these records, since the sensors of the recorder are installed at 2.1 m below the sea level. Profile 2 is the back computed sea state after applying the attenuation correction for the full frequency range (0.05 to 0.33 Hz) which resulted in an increase of 35% in the variance mainly above 0.12 Hz. The probability density distributions (Fig. 3) follow Normal law very narrowly for these two profiles (see Table 1). For profiles 3 to 7 the autospectral band used in Eq. (1) is the same (0.05 to 0.12 Hz) which results in a loss of 20% variance from profile 2; but the ITUNE values differ. Accordingly, the profiles show different characteristics (Table 1). For all the profiles (nos. 3-7) the skewness (α_1) estimates vary only marginally but the kurtosis (α_2) values vary widely

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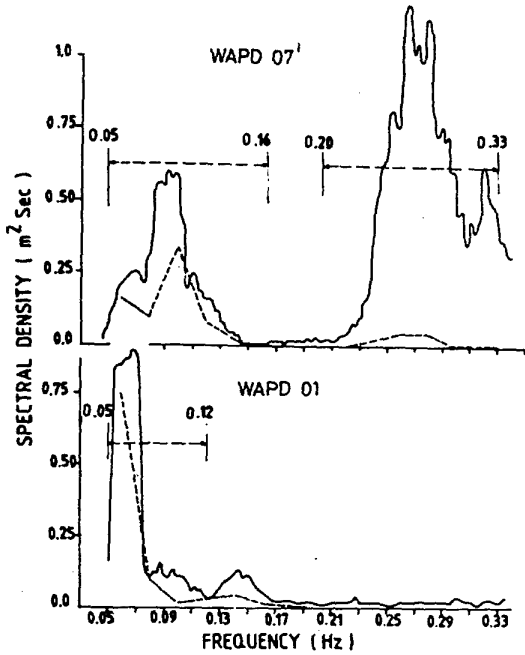


Fig. 1 - Different autospectra, recorded (----) and attenuation corrected (—), with filtered sea and swell regimes used for the generation of the wave profiles.

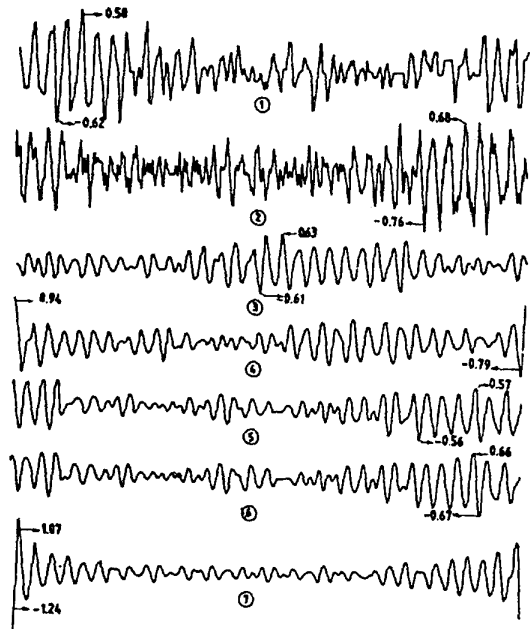


Fig. 2 - A set of wave profiles computed using the tuned swell spectrum (0.05-0.12 Hz) from the sea sta. (WAPD01) (Profile numbers are same as in Table 1)

Table 1 - Statistical and other details of recorded and computed wave profiles

Profile No.	Spect. Tuning (Hz)	Phase Tuning (ITUNE)	[Mean value for all profiles is 0]					Deg. of Fr. (D.F.)	Okay (at % lev)
			Variance (m ²)	Skewness (α_1)	Kurtosis (α_2)	Chi. Sq. (χ^2)			
WAPD01									
1	0.05-0.33	Recorded	0.044	-0.141	3.161	9.3	6	10	
2	"	5	0.060	0.012	3.313	13.9	7	5	
3	0.05-0.12	1	0.048	0.010	2.864	4.3	7	70	
4	"	2	"	0.070	3.876	9.4	7	20	
5	"	3	"	0.008	3.102	20.2	5	N	
6	"	5	"	0.003	3.306	17.8	7	1	
7	"	6	"	-0.096	9.353	68.4	6	N	
WAPD07									
8	0.05-0.33	Recorded	0.043	0.052	2.869	14.1	7	5	
9	"	5	0.191	0.106	2.864	9.7	15	80	
10	"	1	"	-0.054	2.656	22.2	14	5	
11	"	2	"	0.619	13.530	19.1	13	10	
12	"	3	"	0.039	4.164	22.3	16	10	
13	"	6	"	-1.526	32.534	136.2	16	N	
14	0.20-0.33	1	0.141	0.000	2.641	10.3	12	60	
15	"	2	"	0.026	8.310	13.2	10	20	
16	"	3	"	0.008	5.212	30.5	13	N	
17	"	5	"	0.002	2.431	15.5	10	10	
18	"	6	"	-0.042	18.938	117.2	15	N	
19	0.05-0.16	1	0.048	0.037	2.786	4.4	7	70	
20	"	2	"	0.245	8.071	30.7	6	N	
21	"	3	"	0.036	2.698	6.8	6	30	
22	"	5	"	0.003	2.819	6.2	6	30	
23	"	6	"	-0.542	24.652	179.6	7	N	

*N = not okay even at 0% level

from 2.86 to 9.35. Two profiles (5,7) do not follow the Normal law. One very interesting feature noted is the disagreement and agreement of profiles 5 ($\alpha_2 = 3.1$) and 4 ($\alpha_2 = 3.88$) from and with Normal law, respectively. For profile 5 the phase spectrum is given coherency (equal phase values) from the spectral peak (0.068 Hz) to 'spectral peak + half of half energy width' (0.0074 Hz) involving only four component waves and for profile 4 the phase spectrum is assigned computed pseudorandom (Normal) series with mean (=0) and standard deviation (=1). The high α_2 (=3.88) value noted for profile 4 falls much outside the reported range (0.43 to -0.45) by Kinsman¹⁰ while the α_2 value of 3.1 (profile 5) is well within the range, even though, this profile does not follow Normal law. No reasons are evident for this anomalous behaviour. Profile 3 with computed pseudorandom (Uniform) phase structure shows large differences in modulation characteristics compared to profile 2. As noted earlier profile 3 has a variance drop of 20% from profile 2, due to high frequency filtering. Table 1 also shows that the χ^2 value for profile 3 is far better than that for profile 2. Profiles 2 and 6 compare favourably in modulation features and in high wave groups as their phase structures are the same (observed). But profile 2 does not contain the high frequency components. Profile 4 (with Normal phase spectrum) and profile 7 with running averaged (of recorded phase spectrum) compare well in occurrences of terminal high waves and modulation features. Profile 7 shows smooth modulation and huge wave heightening with a 2.31 m wave whereas the observed maximum (profile 1) is only about 1.2 m. The Normal fits for all the profiles are shown in Fig. 3 and their distribution parameters are listed in Table 1 for comparative evaluation.

Effect on tuned swell (dominant sea part filtered)—Fig. 4 shows the computed wave profiles with different phase spectral tunings on a secondary swell peak (~20% variance) after removing the dominant sea (~80% variance) peak (Fig. 1). In this group only two profiles (20,23) do not follow Normal law (see Fig. 5 also), and have Normal and running averaged phase spectra, respectively. For both these profiles the α_2 values are abnormally high (8.07 and 24.65). The skewness value (-0.54) for profile 23 is very much outside the range of Kinsman's values¹⁰. These profiles show similar features as those of profiles 4 and 7 in Fig. 2. The wave heightening of these two profiles, compared to profile 19, is remarkable from ~1.25 m to 3.18 m. Wave group formation is

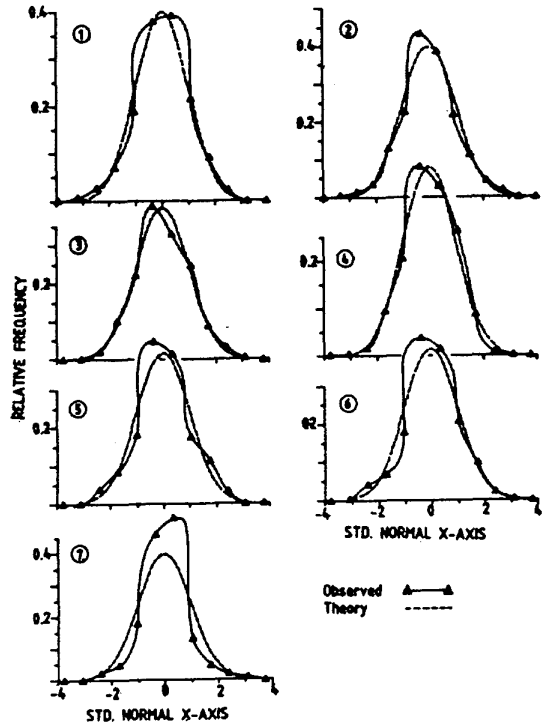


Fig. 3—Normal distribution fits for the surface profiles shown in Fig. 2. X-axis is graduated in standard Normal units (Profile numbers are same as in Table 1)

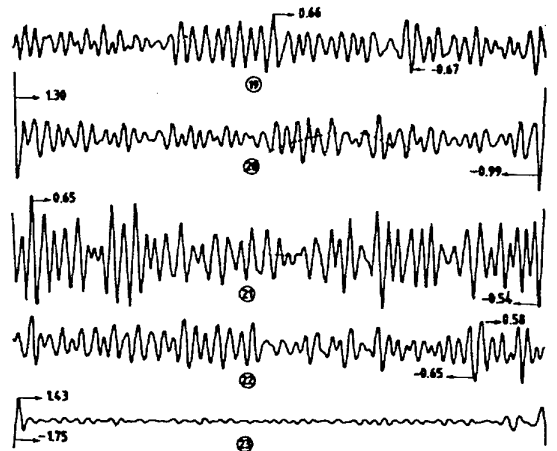


Fig. 4—A set of wave profiles computed using the filtered swell spectrum (0.05-0.16 Hz) from the sea state (WAPD07) (Profile numbers are same as in Table 1)

more in profiles 19, 21 and 22 compared to profiles 20 and 23. Profile 21 is a result of phase coherency at the secondary swell peak involving seven wave components.

The largest variations in χ^2 are noted for running averaged phase tuning (ITUNE=6) both for

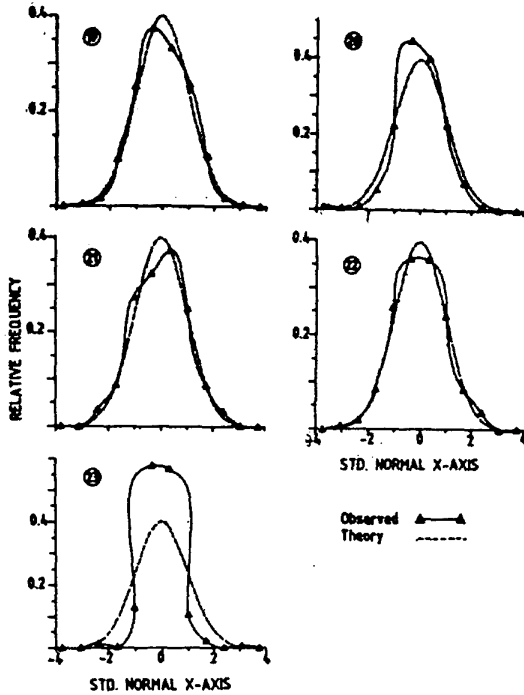


Fig. 5 - Normal distribution fits for the surface profiles shown in Fig. 4. X-axis is graduated in standard Normal units (Profile numbers are same as in Table 1)

WAPD01 and WAPD07 (Figs. 3,5). A noteworthy feature is the behaviour of the profiles 4 and 20, both with pseudorandom (Normal) phase spectra; while profile 4 follows Normal law comfortably, profile 20 does not follow the law. The only reason for this anomalous behaviour is the relative importance of the swell peaks (60 and 20%) in relation to the total variance over the full frequency range (0.05 to 0.33 Hz). Another important feature is the disagreement of profile 5 (for WAPD01) and agreement of the profile 21 (for WAPD07) from and with Normal law, respectively, with phase coherency at the spectral peak (ITUNE=3). The number of wave components assigned phase coherency for WAPD07 is 7 while that in WAPD01 is only 4. A comparison of Fig. 4 with profiles 8 and 9 in Fig. 8 clearly brings out the effect of dominant high frequency filtering.

Effect on tuned dominant sea (secondary swell filtered) - The wave profiles computed with different phase spectral tunings using the high frequency side of WAPD07 as input autospectrum are shown in Fig. 6. The stability of this filtered sea spectrum (0.2 to 0.33 Hz) was checked for breaking instability and is found to be within stable limits. Comparing profiles 17, 16 and 14, it appears

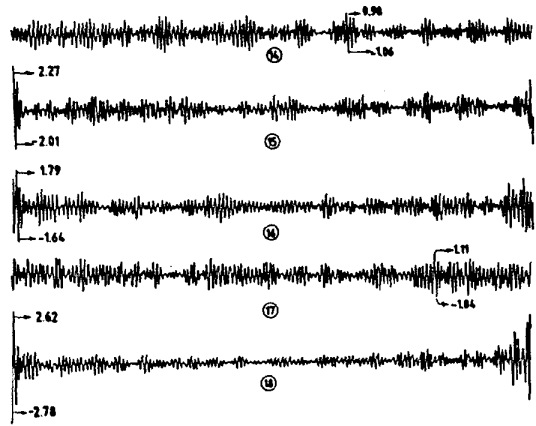


Fig. 6 - A set of wave profiles computed using the filtered sea spectrum (0.2-0.33 Hz) from the sea state (WAPD07) (Profile numbers are same as in Table 1)

that wave group formation is more in profiles 16 and 17. Phase coherency at the spectral peak involving 28 wave components (0.28 to 0.33 Hz) resulted in a height increase (profile 16) of 59% (2.15 to 3.43 m) compared to profile 17 wherein the observed phase spectrum was employed for inverse transform. Profiles 15 and 18 show similar features of modulation with maximum wave heights 4.28 and 5.4 m respectively. The highest wave in profile 18 has an H/L ratio equal to 0.14. It should be noted that in the attenuation corrected full range (0.05 to 0.33 Hz) wave profile (no. 9) the maximum wave height is only about 2.5 m. Hence, a phase dependence in the nature of running averaged phase spectrum resulted in steepening the waves to breaking conditions in the case of a filtered sea spectrum. Out of the 5 profiles only 2 fail in the χ^2 test for Normality; profile 16 with spectral peak coherency and profile 18 with running averaged phase spectrum. Eventhough the other profiles followed Normal law, the probability density curves (Fig. 7) are found to be irregular compared with those of profiles 1-4, 8 and 19.

Effect on multi peaked sea state - Profile nos. 8-13 in Fig. 8 show the effect of phase spectral tuning on a multi peaked sea dominated sea state. Profile 8 is the recorded sea state and 9 is the back computed profile after attenuation correction which resulted in an increase of about 350% in variance from 0.043 to 0.191 m². Profiles 9 and 10 compare well, with the phase spectrum distributed Uniform at 50% confidence level⁸ for profile 9 and computed pseudorandom (Uniform) phase spectrum for profile 10. Running averaged phase spectrum (profile 13) and Normally distri-

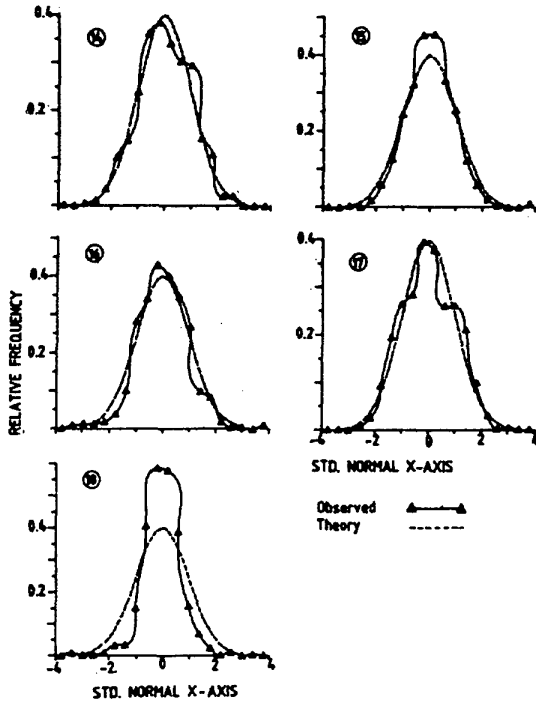


Fig. 7 - Normal distribution fits for the surface profiles shown in Fig. 6. X-axis is graduated in standard Normal units (Profile numbers are same as in Table 1)

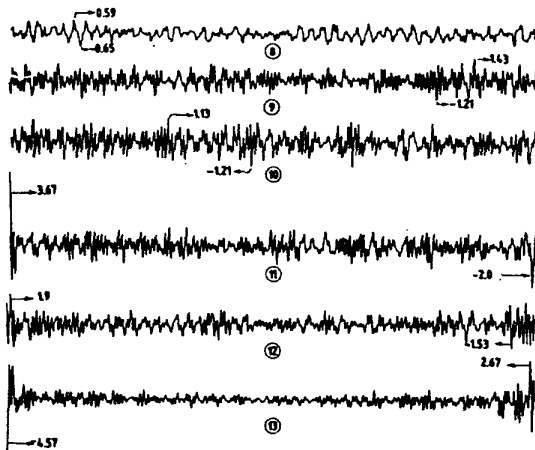


Fig. 8 - A set of wave profiles, recorded and computed using the full range (0.05-0.33 Hz) spectrum of WAPD07 (Profile numbers are same as in Table 1)

buted phase spectrum (profile 11) result in very high steep waves; about 7.0 and 5.7 m respectively. Since these waves have a period of 4 sec (see profiles) their steepness values (H/L) are 0.292 and 0.238, almost double the breaking limit of 0.14. Undoubtedly such abnormal breaking waves would probably never be recorded proper-

ly by the existing instruments and presence of such waves might make the instruments fail or to record spurious marks. Another important aspect is that even in the computed profile the microstructure of the individual crests and troughs are not clear. This would require sampling (or computing) at small intervals, say 0.1 sec or so. Profile 12, computed with phase coherency between 0.28 and 0.33 Hz at the spectral peak involving 28 wave components compares well with the attenuation corrected profile (no. 9). In this group only profile 13 with running averaged phase spectrum does not follow Normal law. Fig. 9 shows the corresponding Normal law fits for profiles shown in Fig. 8.

An interesting feature noted here is the poor fits of the various surface amplitude profiles in relation to Normal probability law. Grouping all the profiles into only 2 categories, namely, single peaked (swell or sea) and double peaked, it is seen that computed pseudorandom (Uniform) phase spectrum gives very good amplitude fits with confidence levels ranging from 60 to 70% for single peaked cases. But in the double peaked (0.05 to 0.33 Hz) case (profile 10) the fit is poor (5%). The fit improves (profile 9) from 5 to 80%

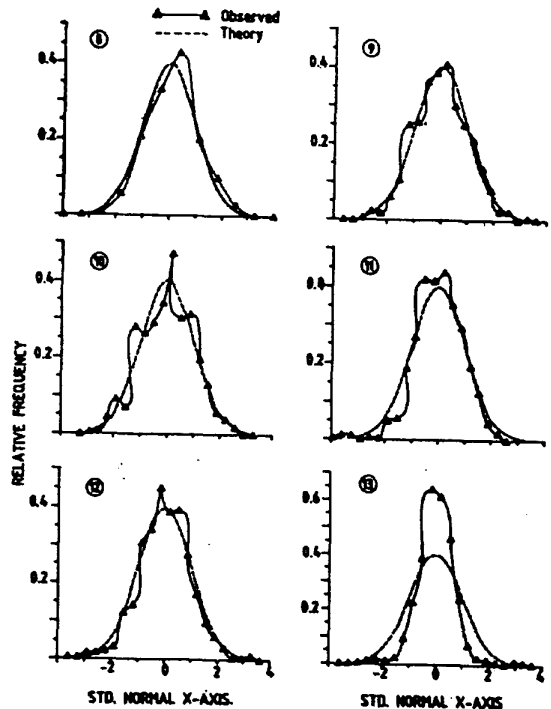


Fig. 9 - Normal distribution fits for the surface profiles shown in Fig. 8. X-axis is graduated in standard Normal units (Profile numbers are same as in Table 1)

when the attenuation corrected autospectrum is used with observed phase spectrum. Varkey⁸ showed that the fits of Uniform law to observed phase spectra had confidence levels from 2 to 75% for a set of 11 records involving narrow band, wide band and multi-peaked sea states. It was also observed⁸ that the running averaged phase spectrum followed Normal probability law with confidence levels varying from 0 to 95%. It is also found by the author (unpublished data) that running averaged series obtained from a pseudorandom Uniform series give only poor fits (0, 1, 2, 10% etc.) even though, theoretically the smoothed series would follow Normal law. Hence, the observed poor fits for the surface amplitudes seem to be agreeable at oceanographic levels.

The skewness estimates of 5 profiles showed values outside the range (0.44 to -0.092) reported by Kinsman¹⁰. The kurtosis estimates of 11 profiles also showed values outside the range (0.43 to -0.45) reported by Kinsman¹⁰. Table 1 shows that excluding the attenuated recorded profile (no. 1) all the other profiles which show abnormal variations (considering Kinsman's ranges as a normal case) are with phase spectra with frequency dependence (ITUNE = 2, 3 and 6). While the skewness values of these profiles show small variations from Kinsman's range, the kurtosis estimates show very large increases up to 32.5 (profile 13); the same profile shows the maximum skewness variation also. It is to be noted that it is a multi-peaked spectrum. Another important aspect noted is that profiles with skewness and kurtosis values within narrow ranges give rise to Normal fits with very variant confidence limits (e.g., profiles 2 and 3; 8 and 9; 8 and 19; 3 and 22) suggesting that abnormal skewness and kurtosis alone do not always point to non-Normal fits in the case of wind waves.

While the use of pseudorandom (Uniform) phase spectrum (ITUNE = 1) gives very good fits for all the narrow band sea states (profiles 3, 14 and 19) the same does not generate a good Normal surface profile (no. 10) for the multi-peaked spectrum. At the same time, the sea state follows the Normal probability law at 80% confidence for profile 9, for which attenuation corrected spectrum and observed phase spectrum are used. Hence, narrow band autospectrum with computed pseudorandom (Uniform) phase spectrum and double peaked autospectrum with observed phase spectrum give very good surface profile fits.

In the case of narrow band swells (profiles 5, 21) it is found that wave groupiness is visually

larger when there exists phase coherency at the spectral peak involving a few number of wave components. For single peaked 'sea' state (profile 16) also there is a mild tendency to larger groupiness when there is coherency at the spectral peak. In the case of multi-peaked sea state (profile 12) no noticeable differences exist. Hence, it seems that a combination of phase coherency at spectral peak and higher peak periods gives rise to increased wave groupiness. The result also suggests that Lake and Yuen's¹ nonlinear wave model might suite well the sea states with large wave groupiness.

Computed pseudorandom (Normal) phase spectrum and running averaged (from observed) phase spectrum always give rise to one or two very high waves (freak waves) in all the cases; i.e., single peaked and multi-peaked. In the case of multi-peaked and 'sea' spectra (Figs. 6, 8) these high waves are very unstable and breaking. Hence, capsizing of boats due to sudden high waves could be a phenomenon due to freak waves caused by a frequency dependence of phase spectrum similar to that assigned for ITUNE = 2 or 6. This type of frequency dependence could be formed in natural situations due to currents and winds. Here it is worth mentioning that Funke and Mansard⁷ considered wind as a necessary factor to 'steepen up' the waves. Now, it could be proposed that the role of wind driven currents might also be important in forming nonlinear waves.

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