

# Chapter 5

## Examples of Horel [1984]

### 5.1 Complex EOF Analysis: Computer Simulated data

We have adopted the procedure given by Horel [1984] for performing Complex EOF analysis in time domain outlined in Chapter 3.5. It would therefore be instructive to also present the results of Complex EOF analysis, obtained by us, using the four “examples” of computer simulated space-time fields discussed by Horel [1984]. The data along with its Hilbert transform for the four examples are shown in Figure 5.1 - Figure 5.4. Just as in Horel [1984] the principal components are computed from the covariance matrix *after* deleting the one tenth of each time series (30 time steps for *Examples 1 – 3*) as well as its Hilbert transform at both the beginning and at the end, so as to avoid “end-effects” associated with Fourier transforms and Hilbert transforms.

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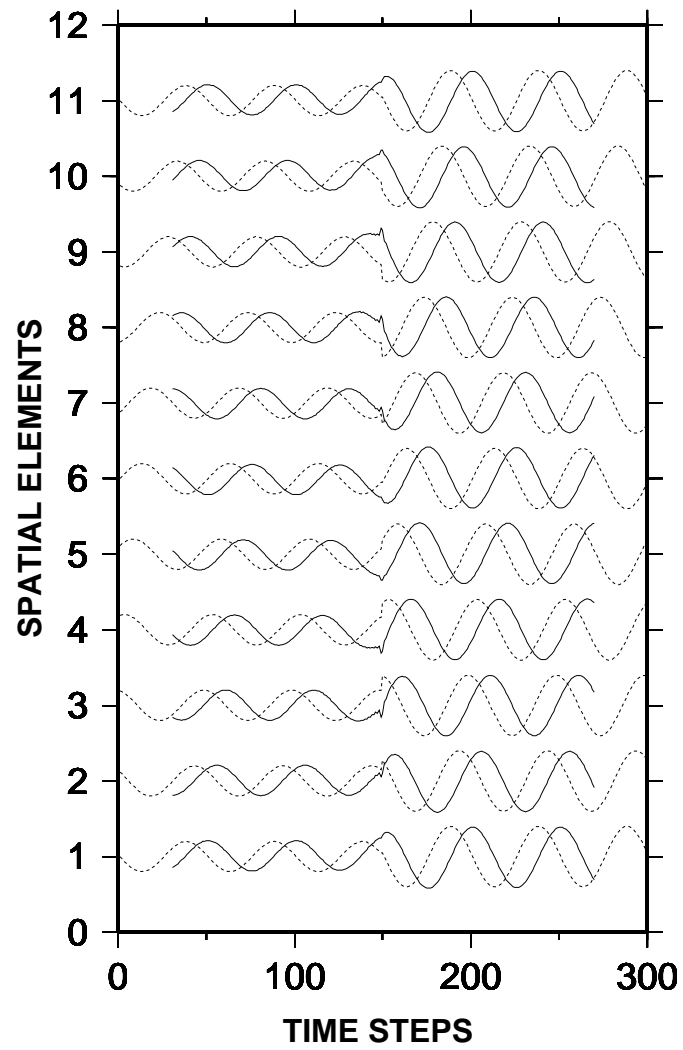
#### 5.1.1 Example 1

*Example 1* of Horel [1984] is of waves travelling 300 time steps over 11 equidistant locations linearly in the direction of the positive  $x$ - axis, i.e., towards the right, having a period of 50 time elements and a wavelength of 10 spatial elements, viz.,

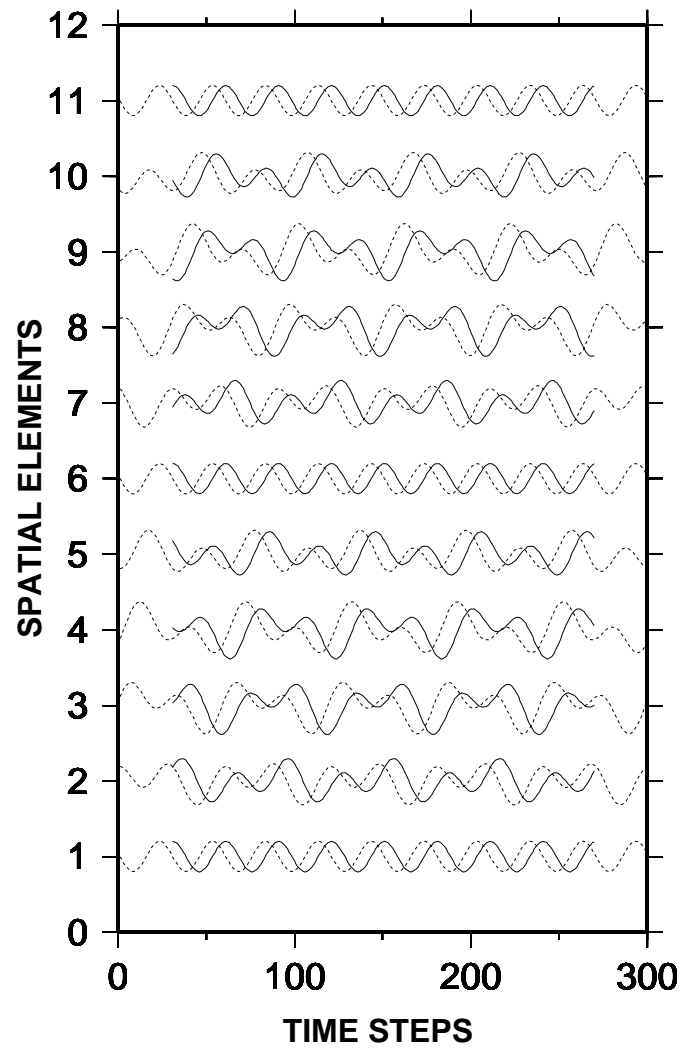
$$u_x(t) = A(t) \sin 2\pi \left( \frac{x-1}{10} - \frac{t-1}{50} \right) \quad (5.1)$$

where  $1 \leq x \leq 11$ ;  $1 \leq t \leq 300$ ;  $A(t) = 1$  for  $t \leq 150$ ,  $A(t) = 2$  for  $t > 150$ . The results of Complex EOF analysis for *Example 1* are shown in Figure 5.5 in *Stick diagram* form. As expected a single spatial principal component along with its associated temporal function explains 100% of the

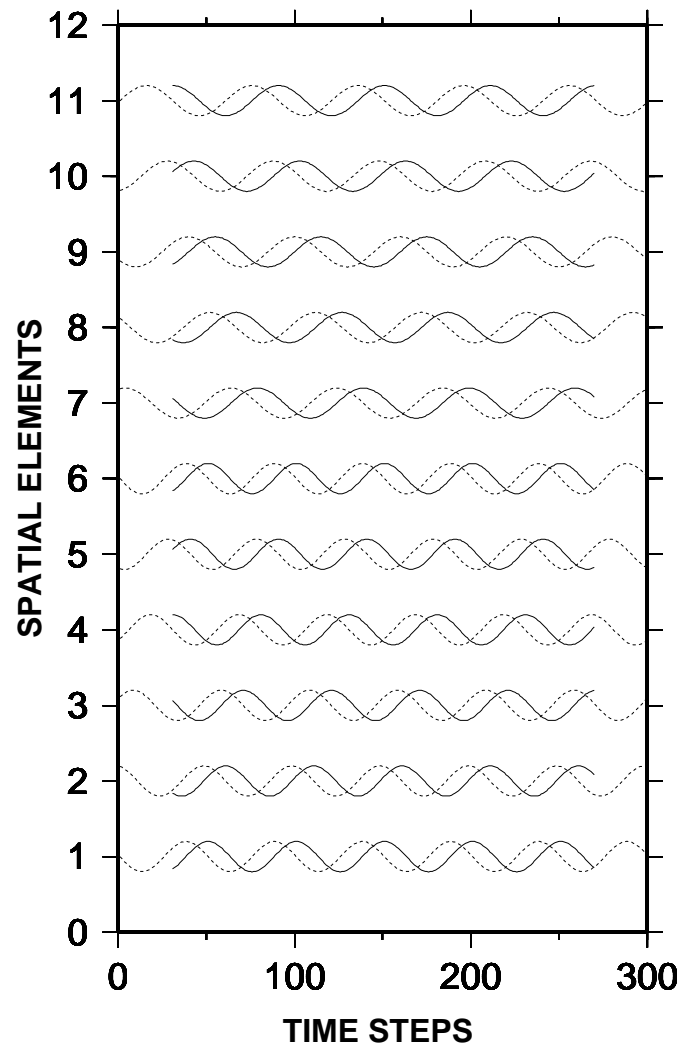
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**Figure 5.1** Example 1 of Horel [1984]: Data (dashed) & Hilbert transform (solid)

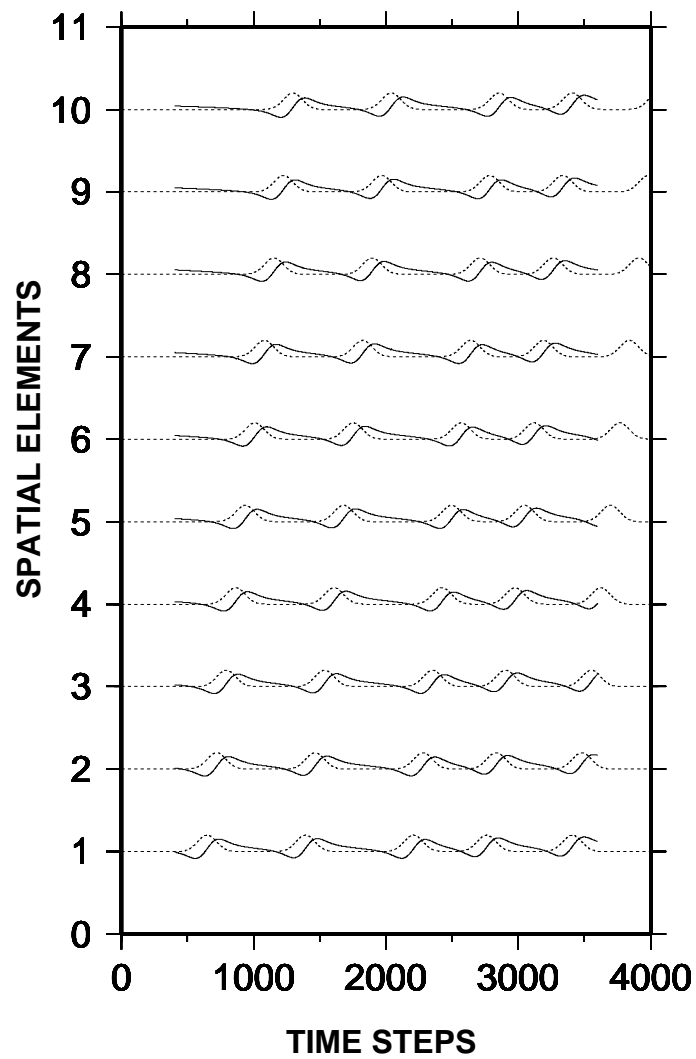
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**Figure 5.2** Example 2 of Horel [1984]: Data (dashed) & Hilbert transform (solid)

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**Figure 5.3** Example 3 of Horel [1984]: Data (dashed) & Hilbert transform (solid)

**Figure 5.4** Example 4#: The example adapted from Merrifield and Guza [1990]. Data (dashed) & Hilbert transform (solid)



variance in the data. Note also that the temporal amplitudes on the right are twice those on the left – as expected. The results of Complex EOF analysis for *Example 1* are presented in Figure 5.6 in *Amplitude & phase diagram* form.

### 5.1.2 Example 2

*Example 2* of Horel [1984] is of a propagating wave, propagating towards the right superimposed with a standing wave, viz.,

$$u_x(t) = \sin 2\pi\left(\frac{x-1}{5} - \frac{t-1}{30}\right) + \sin 2\pi\left(\frac{x-1}{10}\right) \sin 2\pi\left(\frac{t-1}{60}\right) \quad (5.2)$$

The results of Complex EOF analysis for *Example 2* are presented in Figure 5.7 in *Stick diagram* form. The parameter  $v$  defined by (2.27) was found to be equal to 32, which is less than 40, which as discussed in Chapter 2.5 implies that in this case varimax rotation is not appropriate. Therefore the original principal components should be used for interpretation. We note that the first principal component, which explains 69% of the variance of the data has the signature of a progressive wave; while the second principal component, which explains 31% of the variance, has the signature of a standing wave. The results of Complex EOF analysis for *Example 2* are presented in Figure 5.8 in *Amplitude & phase diagram* form.

### 5.1.3 Example 3

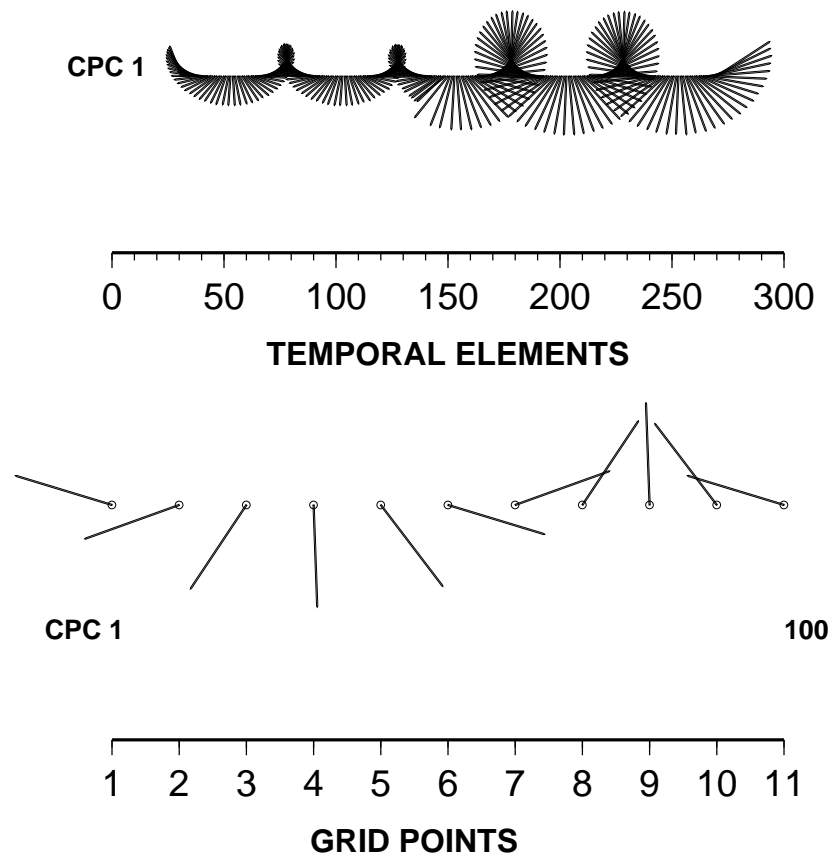
*Example 3* of Horel [1984] is of a propagating wave, propagating towards the right in the left-hand half of the domain and propagating towards the left in the right-hand half of the domain.

$$u_x(t) = \sin 2\pi\left(\frac{x-1}{5} - \frac{t-1}{50}\right) \text{ for } 1 \leq x \leq 6; \quad u_x(t) = \sin 2\pi\left(\frac{x-1}{5} + \frac{t-1}{60}\right) \text{ for } 6 < x \leq 11 \quad (5.3)$$

The results of Complex EOF analysis for *Example 3* are presented in Figure 5.9 in *Stick diagram* form. The parameter  $v$  defined by (2.27) was found to be equal to 93, which is greater than 60, which as discussed in Chapter 2.5 implies that in this case varimax rotation is *indeed* appropriate. Therefore the rotated principal components should be used for interpretation. We note that the first rotated principal component, which explains 54% of the variance of the data has the signature of a progressive wave propagating to the right and has zero amplitude in the right-hand half of the domain; while the second rotated principal component, which explains 46% of the variance, has the signature of a propagating wave propagating to the left and has zero amplitude in the left-hand half of the domain as expected. The results of Complex EOF analysis for *Example 3* are presented in Figure 5.10 in *Amplitude & phase diagram* form - the rotated principal components only are shown. **Note that the spatial phase increases in the direction of propagation!** The temporal phase always increases with time, immaterial of the direction of propagation.

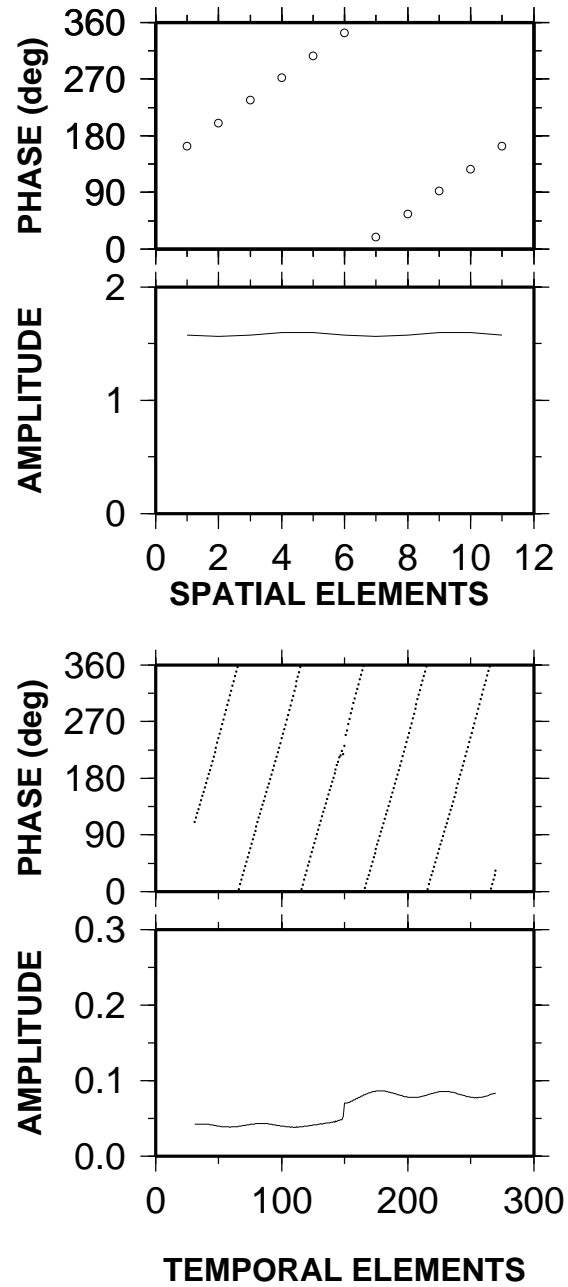


**Figure 5.5** Complex EOF analysis: Example 1. *Stick diagram*. Temporal function (top); Spatial function (bottom), the figure to the right indicates the percentage of variance explained by the principal component

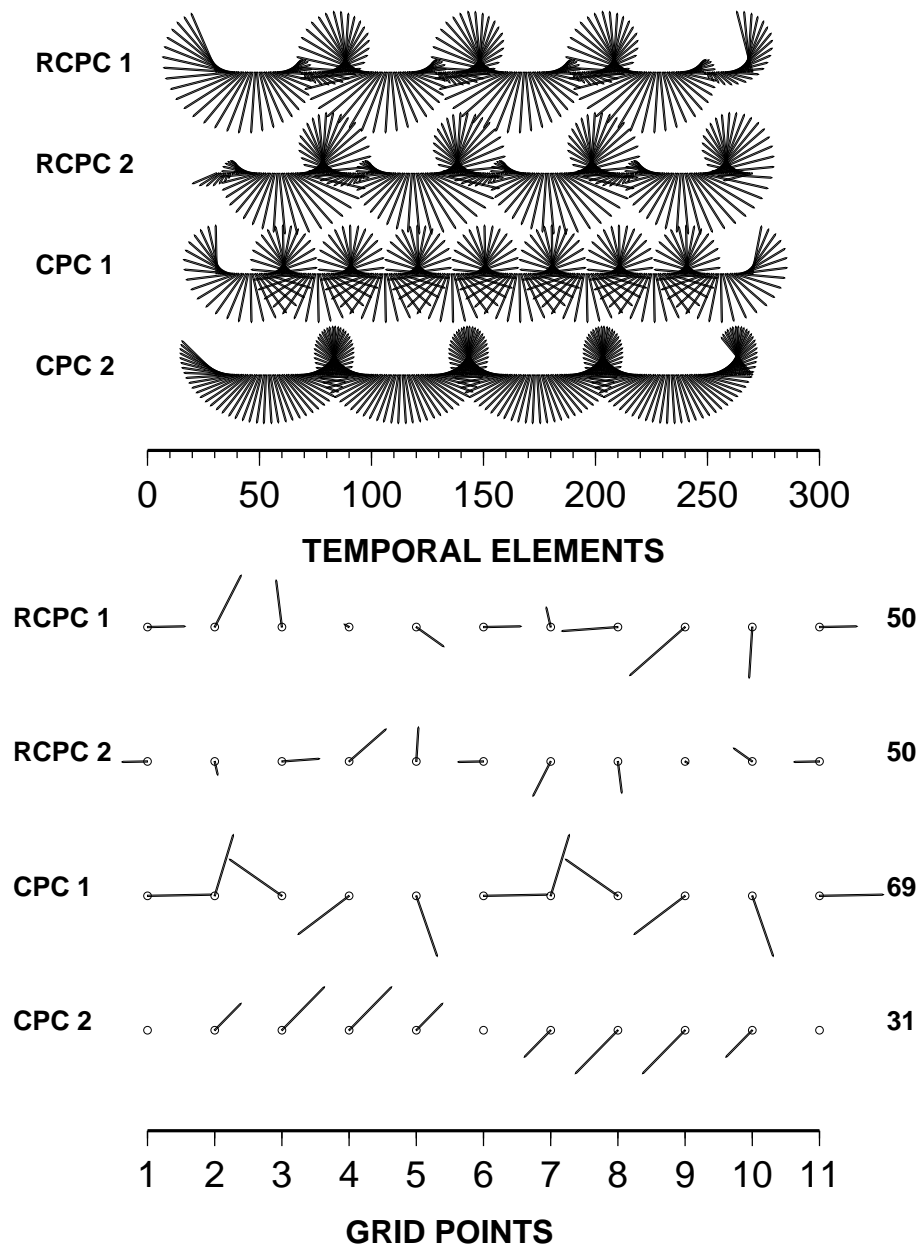




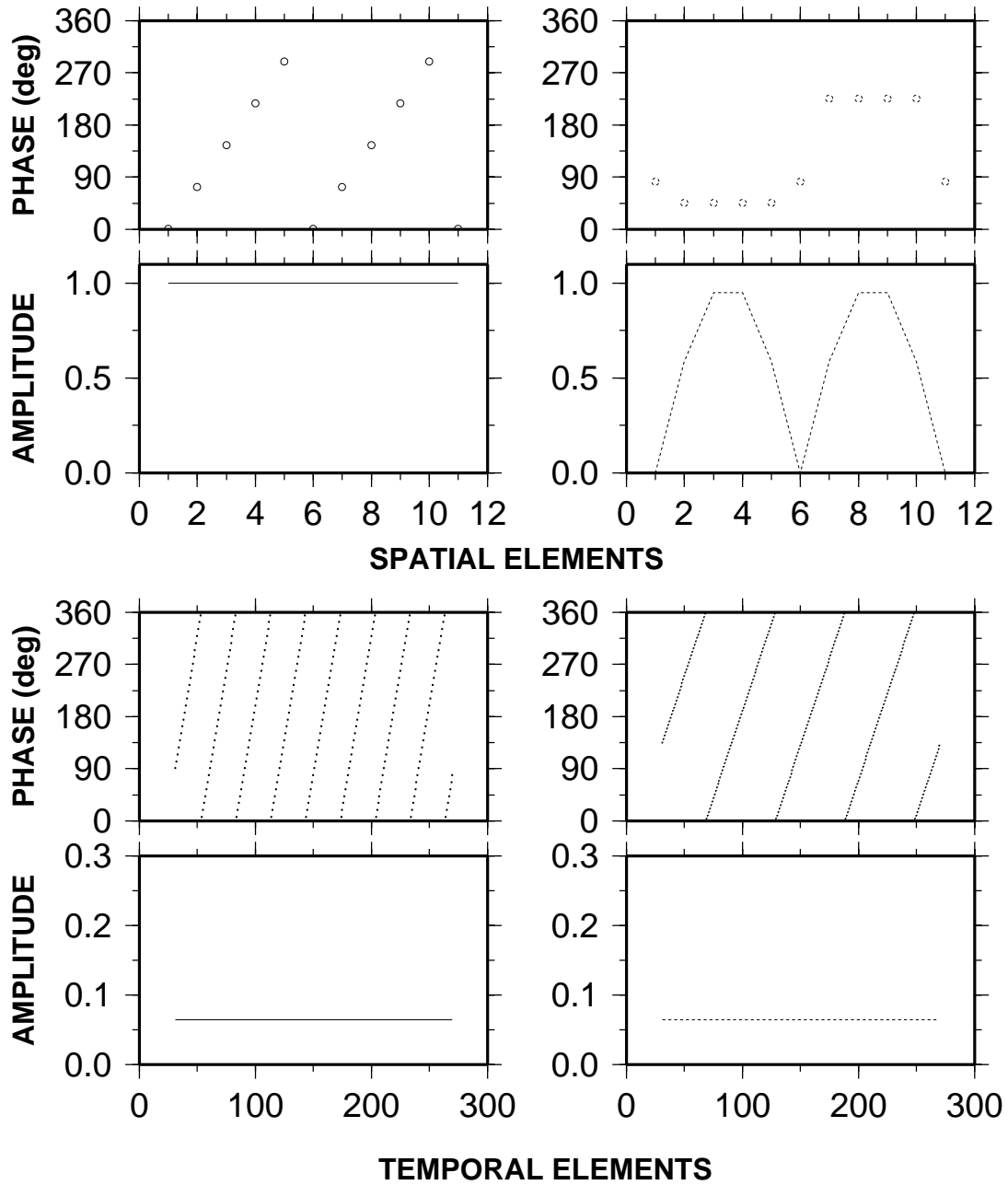
**Figure 5.6** Complex EOF analysis: Example 1. *Amplitude & phase.* Spatial functions (top), Temporal functions (bottom)



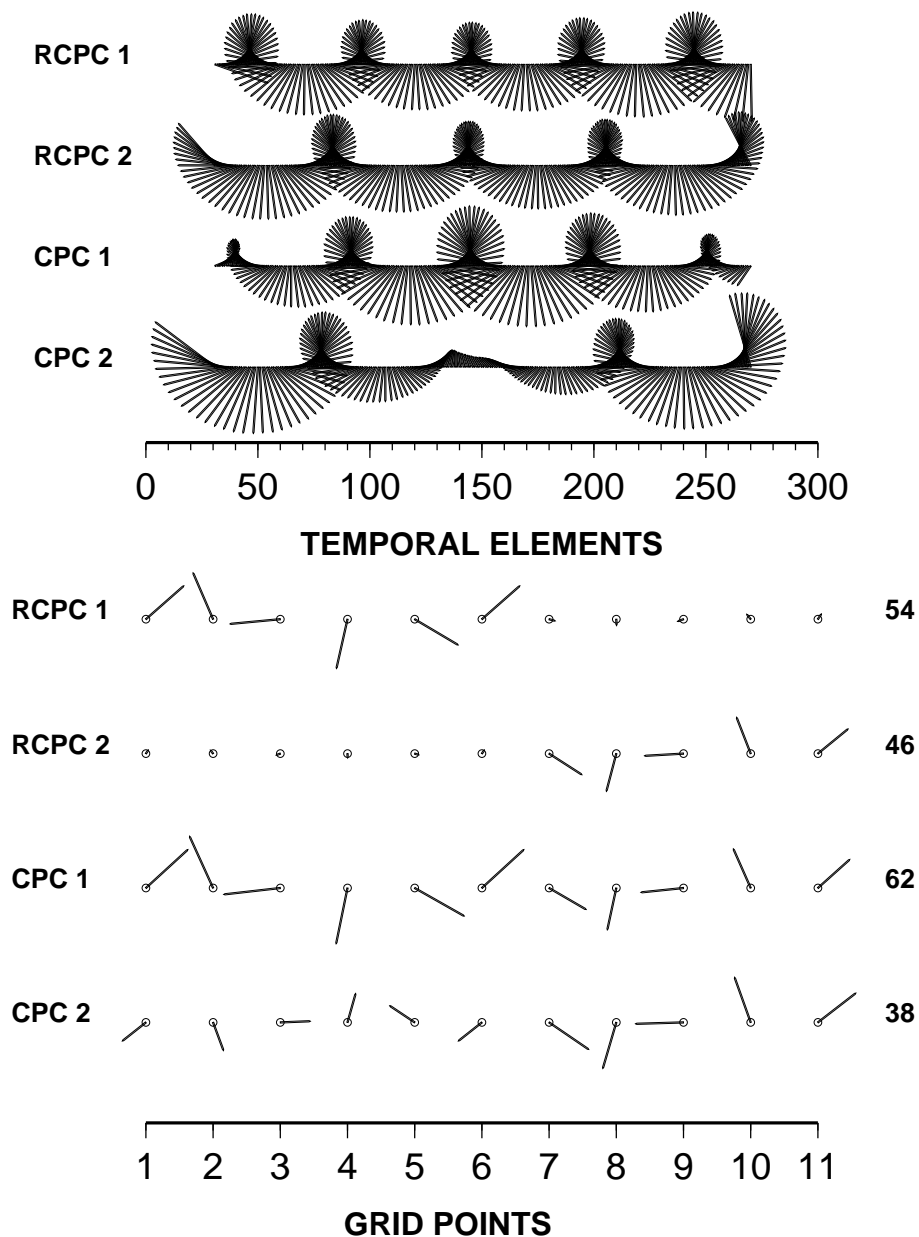
**Figure 5.7** Complex EOF analysis: Example 2. *Stick diagram*. Temporal functions (top); Spatial functions (bottom), the figures to the right indicate the percentage of variance explained by the principal components.  $v = 32$ , which is less than 40, indicating that the unrotated principal components are better!



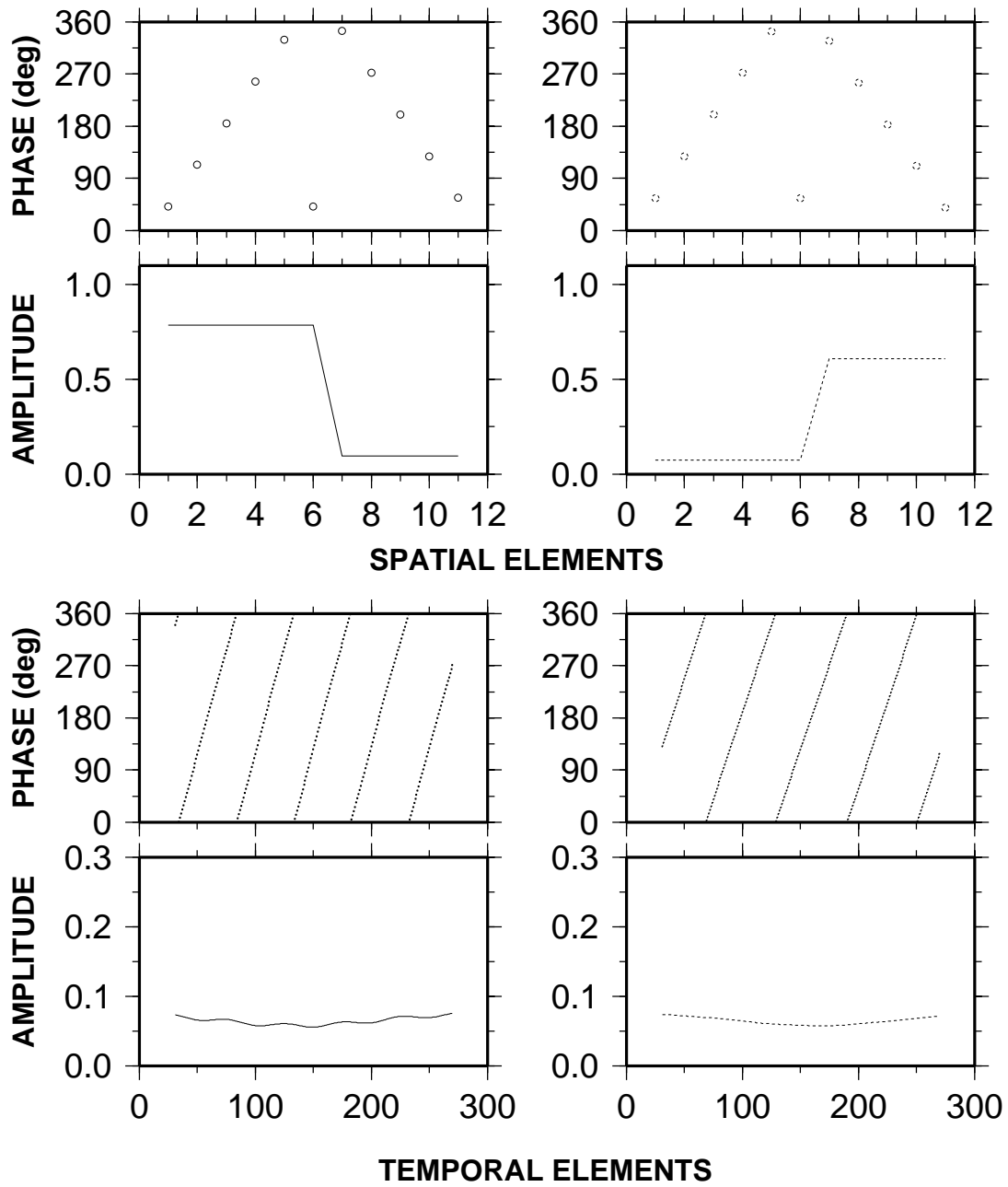
**Figure 5.8** Complex EOF analysis: Example 2. *Amplitude & phase*. Spatial functions (top), Temporal functions (bottom); First principal component (left), Second principal component (right)



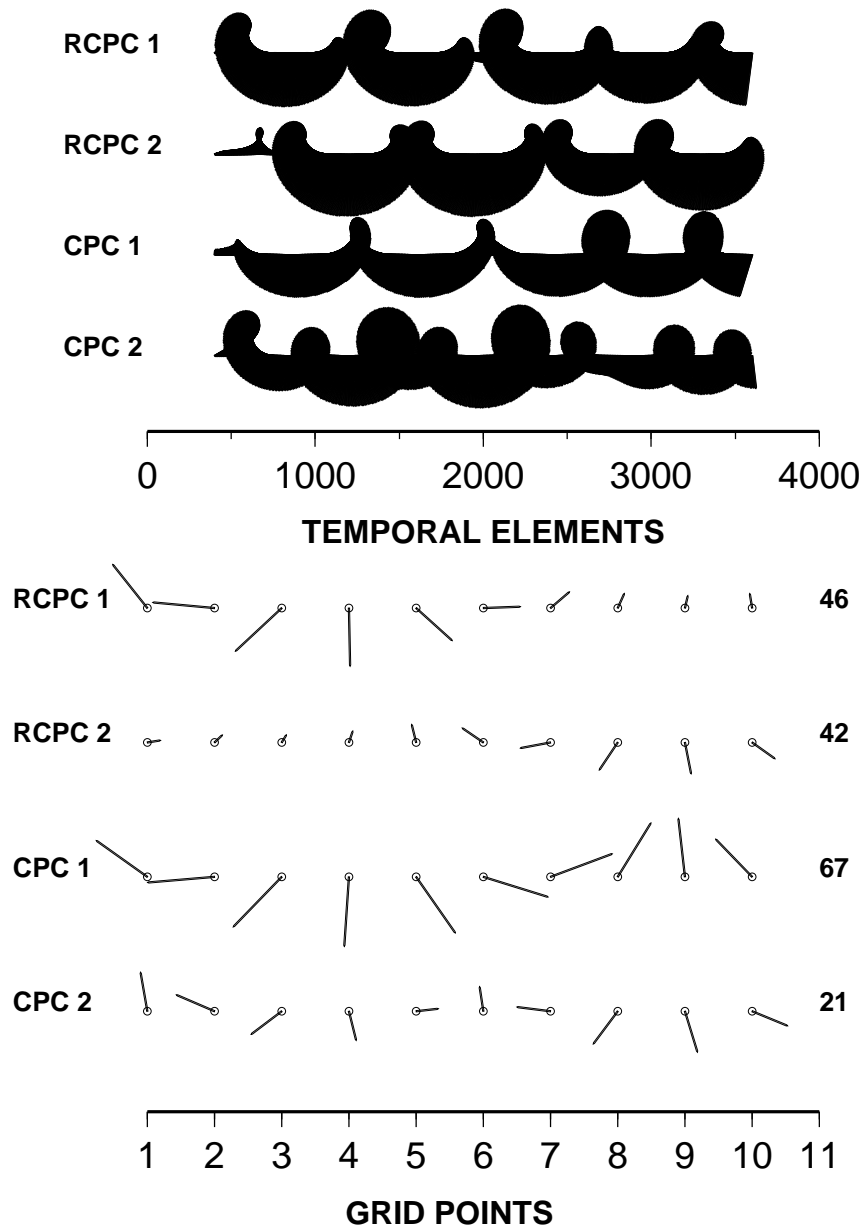
**Figure 5.9** Complex EOF analysis: Example 3. *Stick diagram*. Temporal functions (top); Spatial functions (bottom), the figures to the right indicate the percentage of variance explained by the principal component.  $v = 93$ , which is greater than 60, indicating that the rotated principal components are better!



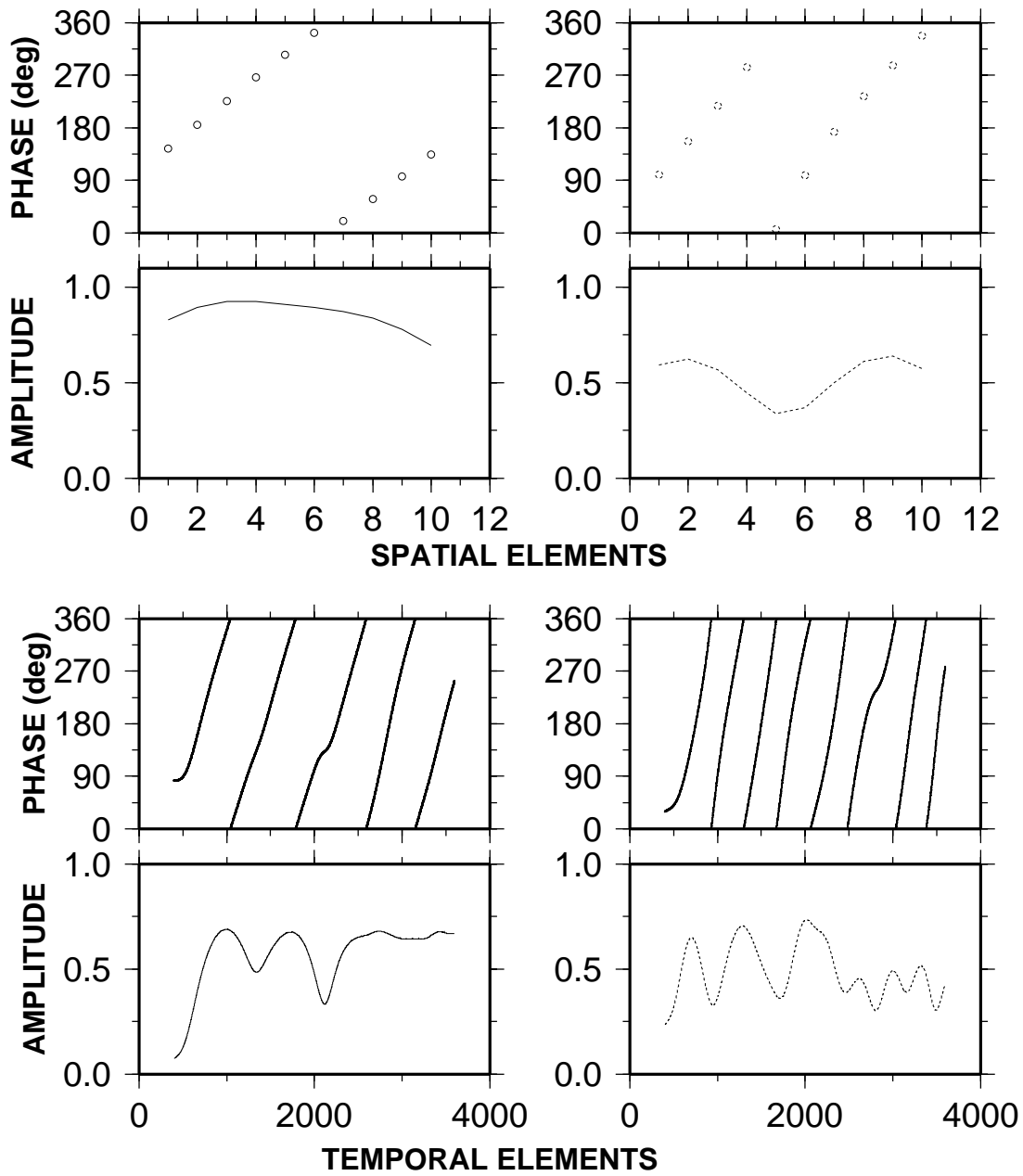
**Figure 5.10** Complex EOF analysis: Example 3. *Amplitude & phase*. Varimax rotated functions. Spatial functions (top), Temporal functions (bottom); First principal component (left), Second principal component (right)



**Figure 5.11** Complex EOF analysis: Example 4#: The example adapted from Merrifield and Guza [1990]. *Stick diagram*. Temporal functions (top); Spatial functions (bottom), the figures to the right indicate the percentage of variance explained by the principal component.  $v = 54$ , which is less than 60, indicating that the rotated principal components need not be considered!



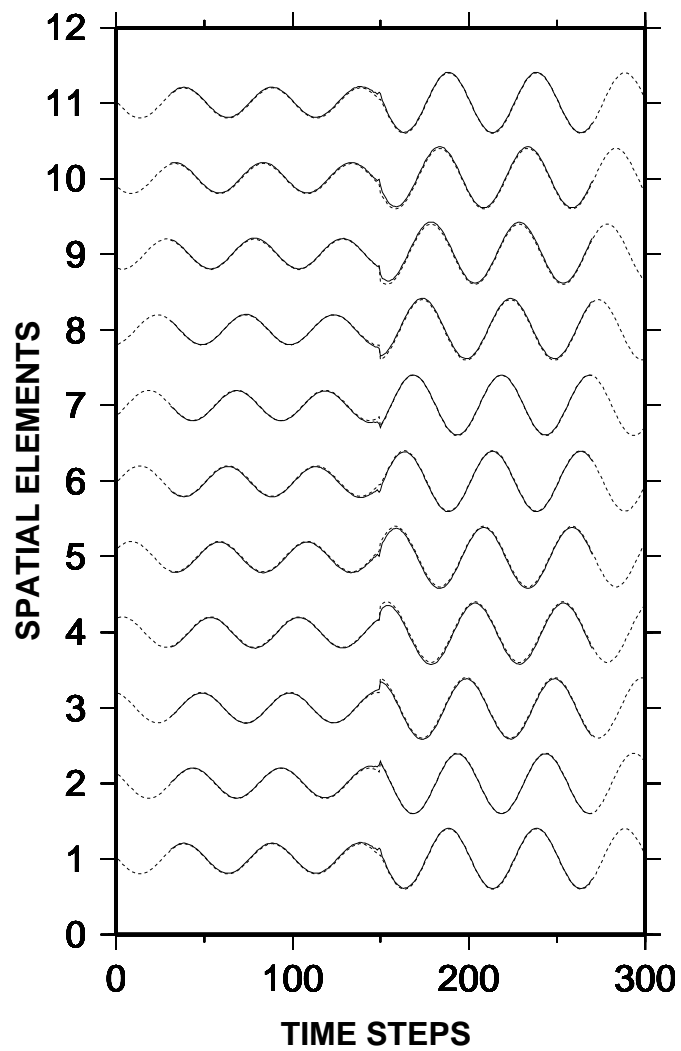
**Figure 5.12** Complex EOF analysis: Example 4#: The example adapted from Merrifield and Guza [1990]. *Amplitude & phase*. Varimax rotated functions. Spatial functions (top), Temporal functions (bottom); First principal component (left), Second principal component (right)



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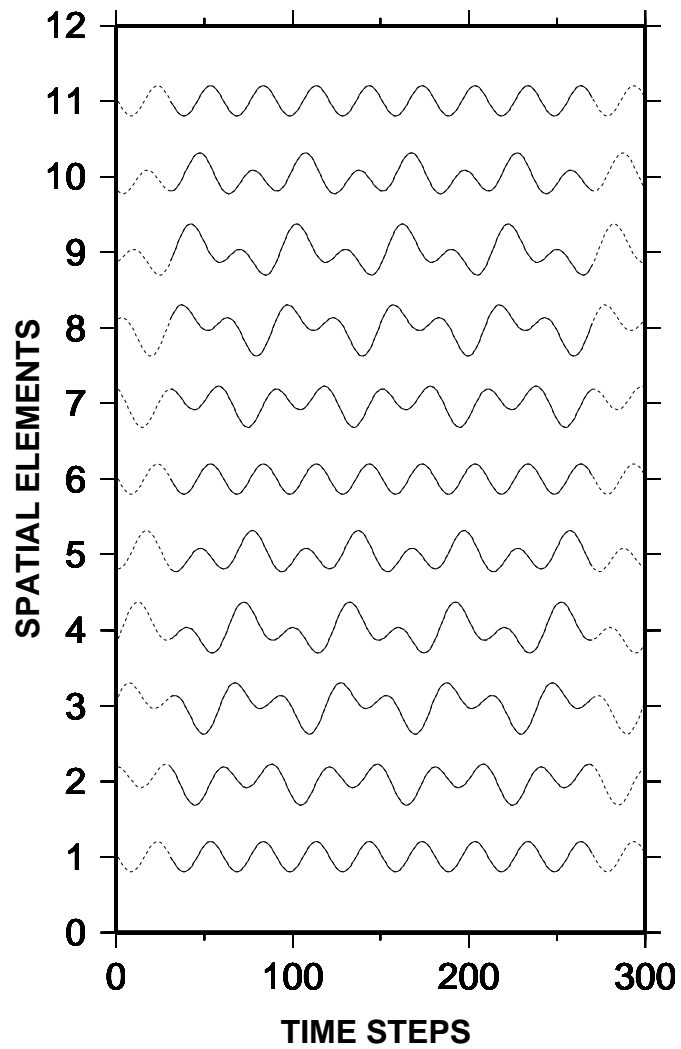
**Figure 5.13** Example 1 of Horel [1984]: Data (dashed) & Reconstructed data from the single pair of spatial and temporal Complex Empirical Orthogonal Functions (solid)

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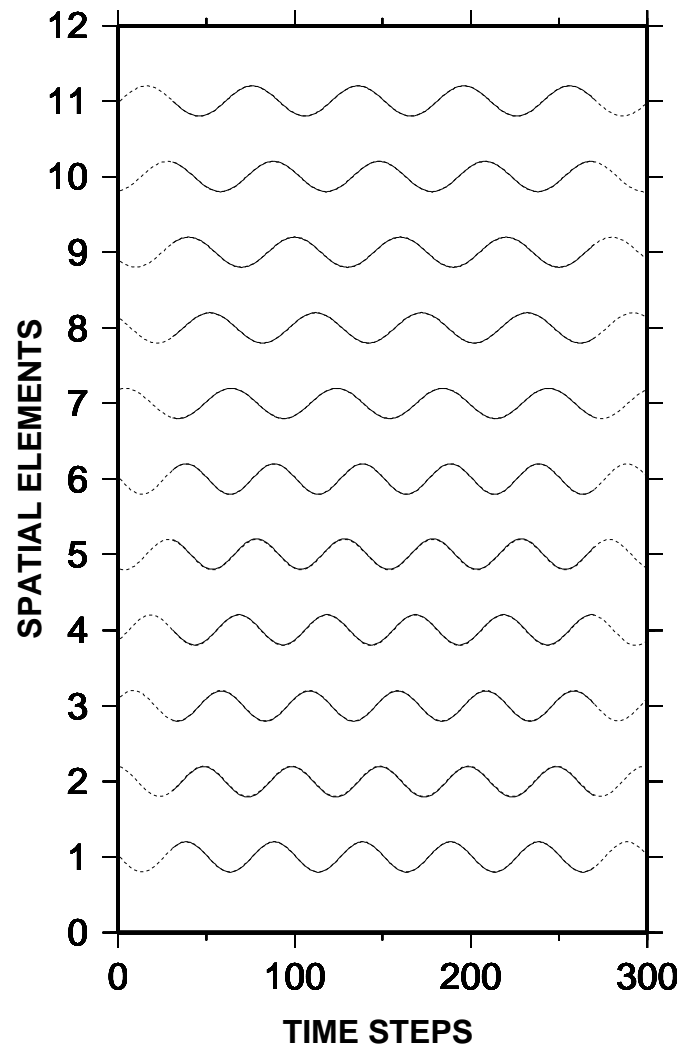
**Figure 5.14** Example 2 of Horel [1984]: Data (dashed) & Reconstructed data from the two pairs of spatial and temporal Complex Empirical Orthogonal Functions (solid)



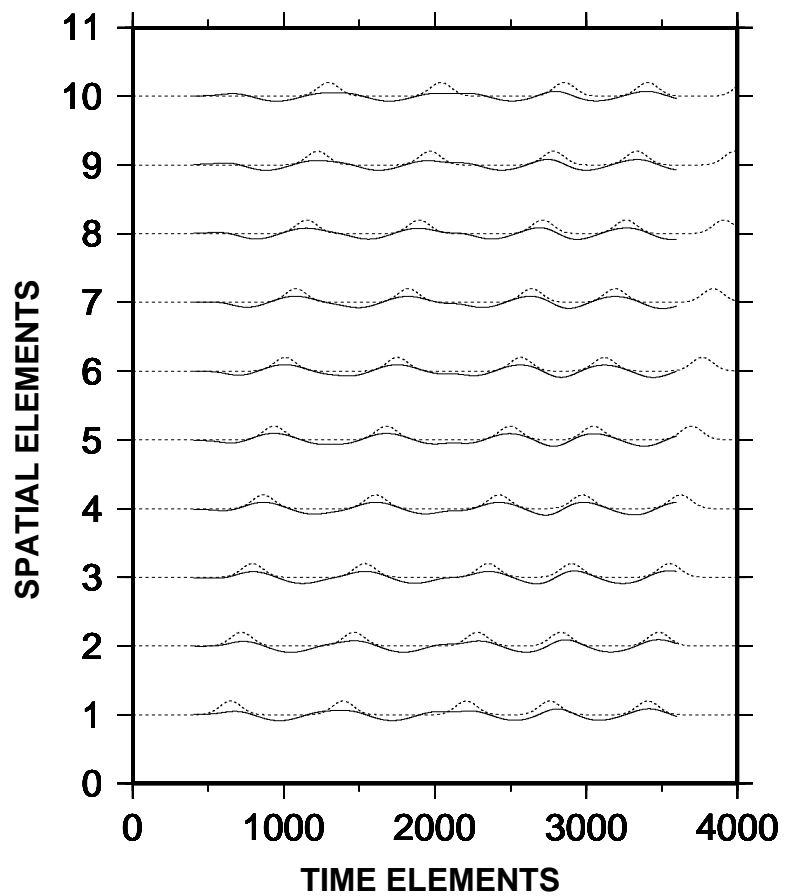
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**Figure 5.15** Example 3 of Horel [1984]: Data (dashed) & Reconstructed data from the two pairs of Varimax rotated spatial and temporal Complex Empirical Orthogonal Functions (solid)

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**Figure 5.16** Example 4#: The example adapted from Merrifield and Guza [1990]. Data (dashed) & Reconstructed data from the first pair of spatial and temporal Complex Empirical Orthogonal Functions (solid)



**Figure 5.17** Example 4#: The example adapted from Merrifield and Guza [1990]. Data (dashed) & Reconstructed data from the two pairs of spatial and temporal Complex Empirical Orthogonal Functions (solid)

