SIGNIFICANCE OF RELATIVE VELOCITY IN DRAG FORCE OR DRAG POWER ESTIMATION FOR A TETHERED FLOAT

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ABSTRACT

There is difference in opinion regarding the use of relative velocity instead of particle velocity alone in the estimation of drag force or power. In the present study, a tethered spherical float which undergoes oscillatory motion in regular waves is considered for the analysis. Float velocity is computed through dynamical equation of motion and particle velocity using linear wave theory. The method of computation is briefly described. The results show that variation of particle velocity with respect to wave period, wave height or water depth is small compared to the variation of float velocity with respect to the same parameters. The results further indicate that the difference between float velocity and particle velocity is considerable and in such cases, relative velocity instead of particle velocity has to be considered for drag force or drag power computation. It is suggested that float velocity must be at least twice the particle velocity in order to use relative velocity in drag force or drag power estimation.

INTRODUCTION

The need for accurate estimation of hydrodynamic forces exerted on a floating sphere, describing harmonic oscillations under or on a free surface. Ade and Martin (1974) treated the two-dimensional problem of moored floating objects in deep water using John’s method. In all these investigations, only freely floating bodies are considered as an approximation to slackly moored conditions. Generally, drag force is ignored in the equation of motion for the following two reasons: (i) most of the offshore structures are large in size and drag force is almost negligible, and (ii) omission of drag force simplifies the solving procedure. Garrison (1978) developed his analysis using the method of distributed sources with allowances made for viscous effects when small diameter members give rise to flow separation. He stated that if viscous effects are disregarded, the results of the potential flow theory approach those based on the inertia term in the Morison equation. This shows that the drag term can be incorporated in the potential flow theory if found significant. Even though the total fluid forces may not be affected significantly by viscous and vortex shedding effects, these may contribute appreciably to the damping, which is the major contributor for wave energy attenuation. Seymour and Isaac (1974) and Agerton et al (1976) have studied the wave attenuation performance of the tethered float breakwater, assuming that the drag produced during the rapid oscillation of the floats is the major contributor for the dissipation of wave energy.

Since there is difference in opinion regarding the use of relative velocity instead of fluid particle velocity in estimating drag force for moving objects (Brebia & Walker, 1979 and Vughts, 1979), in the present study both float and fluid particle velocities are individually found out to compare their magnitudes. Dunwoody and Vandiver (1987) are also of the opinion that if the structure as well as the fluid is in motion, particle velocity may be replaced by the relative velocity.

METHOD OF ANALYSIS

Ocean waves are random in nature. However, it has been shown by Milgram (1976) that the difference between the actual wave profile and the sinusoidal approximation is very marginal since most ocean waves have steepness less than 0.10. Hence, linear wave theory is used to compute fluid particle velocity. Horizontal particle velocity is given as,

\[ u = \frac{nH \cosh k(z + d)}{T \sinh kd} \cos(kt - \omega t) \]

where, 
- \( H \) = Wave height
- \( T \) = Wave period
- \( k \) = Wave number
- \( d \) = Water depth
The magnitude of particle velocity obtained at 16 equal intervals over a wave cycle is used for estimating average particle velocity. The suitability and satisfactory accuracy in adopting linear wave theory for practical applications have been confirmed by the studies of Hogben et al. (1977). The float displacement and subsequently the float velocity are found out using potential flow theory, incorporating linearised drag force and added mass and damping coefficients determined from the motion generated velocity potential in the linear equation of motion (Vethamony, 1990).

For the analysis, a single tethered float which undergoes oscillatory motion in sinusoidal waves is considered. As the wave energy reduction by the float is mainly due to horizontal motion of the floats, only surge mode of motion is considered. If the fluid is assumed to be inviscid, incompressible and irrotational and the harmonic waves are of small amplitudes, then the surge motion of the body can be considered as harmonic with small amplitude. Therefore,

\[ x(t) = \text{Re}(X_0 e^{i\omega t}) \]

Where,
- \( x(t) \) = Surge displacement
- \( X_0 \) = Corresponding complex amplitude
- \( j = \sqrt{-1} \)
- \( \omega \) = Radian frequency
- \( \text{Re} \) = Real part

The simplified equation of motion for surge is written as,

\[
\begin{align*}
( m + \alpha ) \left( -\omega^2 - (\beta + Q) j\omega + C \right) X_0 \\
= f_w - ( m + \alpha ) \omega^2 \xi_0 - (\beta + Q) j\omega \xi_0
\end{align*}
\]

where,
- \( m \) = Mass of the float
- \( \alpha \) = Added mass
- \( \beta \) = Damping coefficient
- \( Q \) = Drag term
- \( c \) = Restoring coefficient
- \( f_w \) = Amplitude of wave force
- \( \xi_0 \) = Amplitude of particle displacement

Therefore,

\[ X_0 = \frac{f_w - ( m + \alpha ) \omega^2 \xi_0 - (\beta + Q) j\omega \xi_0}{c - \alpha' ( m + \alpha ) - j\omega (\beta + Q)} \]

\( X_0 \) can be written as,

\[ X_0 = X_\infty + X_{\alpha 0} \]

where, \( X_\infty \) and \( X_{\alpha 0} \) are real functions.

The magnitude of \( X_\infty \), say \( a_\infty \) can be obtained from,

\[ a_\infty = \frac{\sqrt{X_{\alpha 0}^2 + X_{\alpha 0}^2}}{} \]

The computational method to determine the amplitude of float displacement is explained by Vethamony (1990). For harmonic motion, float velocity is written as,

\[ x(t) = \text{Re}(e^{j\omega t}) \]

Therefore, amplitude of float velocity is

\[ |x| = \omega x_0 = \omega a_\infty \]

Drag appears as a result of boundary layer separation and the formation of a wake behind the float.

Drag force \( F_D \) is given by,

\[ F_D = 0.5 C_D \rho A |u| \]

where,
- \( C_D \) = Drag coefficient
- \( \rho \) = Sea water density
- \( A \) = Frontal area of the float

As the float velocity is comparable or greater than the fluid particle velocity, it is advisable to replace the particle velocity term in the drag force with the relative velocity \( \ell = x - u \). Therefore,

\[ F_D = 0.5 C_D \rho A |\ell| \]

Relative velocity has got major influence on the calculation of structural response if the structure responds dynamically to the fluid excitation and this excitation is dominated by drag load (Vught, 1979). To include drag force in the linear equation of motion, the non-linear term \( i |\ell| \) is replaced with \( C'_\ell \). Assuming that water particle motion and relative velocity are Gaussian processes, probability density function for the relative velocity can be obtained in terms of standard deviation of the relative velocity \( \sigma_\ell \). Then the linearised term, \( C' \) becomes \( C'_\ell = \sqrt{\frac{\ell^2}{\pi}} \sigma_\ell \). Thus, only the standard deviation of relative velocity is needed to be known to linearise the drag force. Malhotra and Penzen (1970) have given a procedure (iteration method) to estimate the value of \( \sigma_\ell \). The iteration is started with \( \sigma_\tau \), the standard deviation of particle velocity and a cyclic procedure is used to obtain \( \sigma_\ell \). Similarly, Kuchida et al. (1986) expressed \( \sigma_\ell \) as,

\[ \sigma_\ell = \sqrt{\sigma_\ell^2 + \sigma_\nu^2} \]

where,
- \( \sigma_\nu^2 \) = Variance of structural velocity
- \( \sigma_\nu^2 \) = Variance of fluid particle velocity

Determination of \( \sigma_\ell \) involves iteration technique with an appropriately assumed value for \( \sigma_\nu^2 \). The method of Kuchida et al is not as accurate as that of Malhotra and Penzen.
because of the difficulty in assuming an appropriate value for \( a_i^2 \). Using relative velocity drag force is written as,

\[
F_D = 0.5 \, C_D \, \rho \, A \, \sqrt{\frac{g}{\pi}} \, a_i (\dot{x} - \dot{u})
\]

Seymour and Isaacs (1974), conducted experiments with widely differing wave climates and obtained \( a_i \) between 40 and 80 cm/sec with a mean of 56 cm/sec. In the present study \( a_i \) is considered as 0.56 m/sec. Theoretical results with \( C_D = 0.6 \) only are presented here.

Drag power \( (P_D) \) of the float is proportional to the cube of the relative velocity between float and fluid particles. Therefore,

\[
P_D = \mathcal{Q} (|\dot{x} - \dot{u}|) (|\dot{x} - \dot{u}|^3)
\]

where, \( \mathcal{Q} = 0.5 \, C_D \, \rho \, A \, \sqrt{\frac{g}{\pi}} \, a_i \)

RESULTS AND DISCUSSION

The variation of float and fluid particle velocities with wave period, wave height, float size, water depth and depth of submergence of float are shown in Figs. 1(a) to 1(e). The results show that variation of particle velocity with respect to the above parameters is very small compared to the variation of float velocity for the same parameters. The results further indicate that the difference between float and particle velocities is considerable and in such cases, relative velocity instead of particle velocity has to be considered for computing drag force or drag power. The float velocity increases with increase in wave period and wave height and diminishes with increase in float size. No variation in float velocity is observed with water depth or depth of submergence of float. Drag force also follows the same pattern (Figs. 2(a) to 2(a)).

In general, when the size of an object increases, drag force also increases. However, in the present case, with increase in float size, float velocity considerably decreases and since particle velocity is very much less than the float velocity it causes reduction in the drag force. It appears that if the size of the float or floating structure is quite large, it is possible for the system to have less velocity than the particle velocity. This condition depicts that if the size of the floating structure is large compared to wave length, structural velocity is negligible and drag force could even be ignored, which would then be a diffraction problem. It is therefore suggested that float velocity must be at least twice the particle velocity in order to use relative velocity in drag force estimation.

Variation of drag power with respect to wave period, wave height, float size, water depth and depth of submergence of float is shown in Figs. 3(c) to 3(e). As relative velocity increases with increase in wave period and wave height and decreases with increase in float size, drag power also follows the same pattern. No significant variation is found in drag power with changes in water depth, except for a slight variation at shallower depths where water particle velocity depends on water depth.

Drag power, which is the dissipative power of the float, is the most important factor useful for studying the wave attenuation characteristics of a group of tethered floats.

CONCLUSION

In the present study, only a spherical float is considered for studying the significance of relative velocity in drag force and drag power estimations. However, for more generalization, studies should be carried out with objects of different shapes.

APPENDIX : REFERENCES


APPENDIX: NOTATION

\[ \begin{align*}
A & = \text{Frontal area of the float} \\
c & = \text{Restoring coefficient} \\
C_D & = \text{Drag coefficient} \\
F_D & = \text{Drag force} \\
f_w & = \text{Amplitude of wave force} \\
j & = \sqrt{-1} \\
m & = \text{Float mass}
\end{align*} \]

\[ 
\begin{align*}
Q & = 0.5 C_D \rho A \sqrt{\frac{g}{\pi}} a_1 \\
i & = \text{Relative velocity} \\
u & = \text{Horizontal particle velocity} \\
x(t) & = \text{Float displacement} \\
x_0 & = \text{Amplitude of float displacement} \\
\dot{x} & = \text{Float velocity} \\
\alpha & = \text{Added mass} \\
\beta & = \text{Damping coefficient} \\
x_o & = \text{Amplitude of particle displacement} \\
\rho & = \text{Sea water density} \\
i & = \text{Relative velocity between float and fluid particle} \\
\omega & = \text{Radian frequency}
\end{align*} \]

Fig. 1 Variation of particle velocity (u) and float velocity (x) with (a) Wave period, (b) Wave height, (c) float size, (d) Water depth and (e) Depth of submergence of float.
Fig. 2 Variation of drag force ($F_d$) with (a) Wave period, (b) Wave height, (c) Float size, (d) Water depth and (e) Depth of submergence of float.

Fig. 3 Variation of drag power ($P_d$) with (a) Wave period, (b) Wave height, (c) Float size, (d) Water depth and (e) Depth of submergence of float.