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AN ALGORITHM FOR DETERMINATION OF GEODETIC PATH FOR APPLICATION IN LONG-RANGE ACOUSTIC PROPAGATION

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An algorithm for determination of Geodetic path
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Abstract

A computer program has been developed for the construction of geodesic path between two points on the spheroidal surface for application in long range acoustic propagation in the ocean. Geodetic equations have been integrated numerically upto the distance of interest subject to initial conditions and a azimuth (forward problem). Inverse problem has been solved iteratively based on the spheroidal geometry to supplement the initial conditions to the forward problem. The algorithm has been test validated. Program listing along with sample output is also appended.

Introduction

For acoustic propagation studies over long ranges in the ocean, the path traversed by a sound signal between a transmitter and a receiver needs to be modelled to predict the signal arrival time before hand. This is required to select suitable experimental site(s) such that optimum site configuration can be obtained for better signal reception. Also the collection of environmental data along the Geodesic is a pre-requisite for the acoustic modelling of signal trajectory. Though the difference in the range between the great circle and geodesic path may not

be significant, the departure between them may be quite large which is not negligible for long range propagation studies.

Considering the earth to be a sphere, a great circle path represents the shortest distance between any two points on the surface. A great circle path is generated on the surface of the sphere when the sphere is intersected by an imaginary plane such that the terrestrial points under consideration and the center of the sphere are in that plane. Examples of such great circles are the Equator and all meridians. This line is also called a normal section, for it contains the local normal (plumb line) at the terrestrial point. However the actual shape of the earth is not a perfect sphere but an ellipsoid - generated by the revolution of the ellipse about its minor axis. For an ellipsoid the problem of computing shortest path on the surface gets more complicated. It is no longer possible to connect two points with a unique normal section. There will be two such normal sections when the computations are reversed between the points. Since our requirement is to determine the shortest path (which is unique) between two points on the earth, a great circle path will not be sufficient and a geodesic has to be computed. The properties of a geodesic are as follows.

- A line on a surface is a geodesic line, if its osculating plane to the curve contains the normal of the surface.

- It is the shortest connection between two points and has the same angle forward and backward.

- The geodesic curvature is zero everywhere. It is a line produced by continuous layout of a 180° angle by transit.

The geodesic computations involve i) the direct problem and ii) the inverse problem. The direct problem uses the coordinates of the initial point 'A' along with a distance and azimuth to an unknown point 'B' to compute the coordinates of the later. In the inverse problem the coordinates of two points are used to estimate the geodesic distance and azimuth.

In the geodesic problem the role of the inverse problem is to supplement the the initial conditions in the form of azimuth and distance to the direct problem for obtaining the required solution.

Present report covers the mathematical equations involved in the theory and the method of obtaining the solution along with the software. Test runs of the source program have been made with the known check values. In the following section the theoretical formulations are detailed.

Theoretical relations:

a) Direct problem:

At any point on the geodesic, the basic governing equations (Bomford, 1977) of the forward problem are as given below:

$$\frac{d\phi}{ds} = \frac{1}{\mu} \cos \alpha$$

$$\frac{d\lambda}{ds} = \frac{1}{\nu} \sec \phi \sin \alpha$$

$$\frac{d\alpha}{ds} = \frac{1}{\nu} \tan \phi \sin \alpha.$$

where α is the local ray direction or azimuth, clockwise from North, ϕ - the latitude and λ - the longitude and ds - an element of distance along the geodesic (fig. 1).

The above equations are integrated numerically using Runge-kutta-Gills method (Ralston, A. 1960) from the given initial point to the desired distance.

The numerical computations in the form of algorithm for the above equations are:

$$X_{i,j}^4 = f_i (X_1, X_2, X_3, X_4)$$

$$X_i = X_i + ds \sum_{j=1}^4 (Y_i - B_j Z_i) A_j$$

$$Z_i = Z_i + 3 \sum_{j=1}^4 (Y_i - B_j Z_i) A_j - C_j Y_i$$

where $i = 1, 2, 3, 4$; $Y_1 = 1$; $Y_2 = \cos \alpha / \mu$; $Y_3 = \sin \alpha / (\nu \cos \phi)$;

$Y_4 = \sin \alpha \tan \phi / \nu$ and initial conditions of

$$X_1(0) = 0$$

$$X_2(0) = \phi_1 \text{ (latitude at source X - coordinate in radians)}$$

$$X_3(0) = \lambda_1 \text{ (longitude at source Y - coordinate in radians)}$$

$$X_4(0) = \alpha_1 \text{ (source to receiver azimuth angle in radians, clock-wise from north)}$$

The coefficients of A_j , B_j , and C_j are

$$A_1 = 1/2, B_1 = 2, C_1 = 1/2$$

$$A_2 = 1 - \sqrt{1/2}, B_2 = 1, C_2 = 1 - \sqrt{1/2}$$

$$A_3 = 1 + \sqrt{1/2}, B_3 = 1, C_3 = 1 + \sqrt{1/2}$$

$$A_4 = 1/6, B_4 = 1, C_4 = 1/2$$

To perform the above integration upto the range of the receiver, distance between the source and receiver points and the azimuth are unknown. Hence these quantities are to be obtained by first solving the inverse problem i.e. to obtain the geodetic distance and initial azimuth between the source to the receiver locations. With this the coordinates and azimuth the intermediate points along the geodesic are obtained.

b) Inverse Problem:

The known quantities are the geodesic positions $P_1(\phi_1, \lambda_1)$ and $P_2(\phi_2, \lambda_2)$ of two points for which the geodesic distance S_{12} and azimuth α_{12} are to be determined:

The latitudes (ϕ_1, ϕ_2) are reduced to a spherical surface as (β_1, β_2) (Krakiwsky, E.J. and D.B. Thomson, 1974) as given in fig. 2.

$$\beta = \tan^{-1} (1 - e^2)^{1/2} \tan \phi$$

The arc length (σ) between the points is computed using

$$\sigma = \cos^{-1} (\sin \beta_1 \sin \beta_2 + \cos \beta_1 \cos \beta_2 \cos \lambda)$$

The problem is of iterative nature, where λ is not known accurately at the beginning. Hence $\lambda = L$ (differences between longitudes) is to be used as first approximation. Then α_{12} is computed as

$$\alpha_{12} = \sin^{-1} \left[\frac{\sin \lambda \cos \beta_2}{\sin \sigma} \right]$$

Azimuth of the geodesic (α) at the equator is computed from

$$\alpha = \sin^{-1} \left[\frac{\cos \beta_1 \cos \beta_2 \sin \lambda}{\sin \sigma} \right]$$

$$\cos 2\sigma_m = \cos \sigma - \frac{2 \sin \beta_1 \sin \beta_2}{\cos^2 \alpha}$$

$$\cos 4\sigma_m = 2 \cos^2 2\sigma_m - 1$$

$$\cos 6\sigma_m = 4 \cos^3 2\sigma_m - 3 \cos 2\sigma_m$$

Then for the i th iteration the difference $(\lambda - L)_i$ is computed with the following equation.

$$\begin{aligned} (\lambda - L) = \frac{e^2}{2} \cos \beta_0 [A' \sigma_T + B' \sin \sigma_T \cos 2\sigma_m + \frac{C'}{2} \sin 2\sigma_T \cos 4\sigma_m \\ + \frac{D'}{3} \sin 3\sigma_T \cos 6\sigma_m + \dots] \end{aligned}$$

$$\text{where } \beta_0 = \cos^{-1} (\sin \alpha_{12} \cos \beta_1)$$

$$\sigma_T = \sigma$$

$$A' = 1 + \frac{e^2}{4} + \frac{e^4}{8} - \frac{e^2}{8} \sin^2 \beta_0 - \frac{e^4}{8} \sin^2 \beta_0 + \frac{3}{64} e^4 \sin^4 \beta_0 + \dots$$

$$B' = -\frac{e^2}{8} \sin^2 \beta_0 + \frac{e^4}{8} \sin^2 \beta_0 - \frac{e^4}{64} \sin^4 \beta_0 + \dots$$

$$C' = -\frac{e^4}{64} \sin^4 \beta_0 + \dots$$

D' = ...

After this λ is computed as $\lambda = L + (\lambda - L)_i$

This iterative process continues till the following condition gets satisfied

$$| (\lambda - L)_{i-1} - (\lambda - L)_i | \leq 0.00001$$

The final azimuth is computed using

$$\alpha_{12} = \sin^{-1} \left[\frac{\sin \alpha}{\cos \beta_1} \right]$$

and the reverse azimuth $\alpha_{21} = \sin^{-1} \left[\frac{\sin \alpha}{\cos \beta_2} \right]$

Then the distance s_{12} is given by

$$s_{12} = b \left[A \sigma_T - B \cos \sigma_m \sin \sigma_T - \frac{C}{2} \cos 4\sigma_m \sin 2\sigma_T - \frac{D}{3} \cos 6\sigma_m \sin 3\sigma_T - \dots \right]$$

where

b = semi minor axis

$$A = 1 + \frac{k^2}{4} - \frac{3}{64} k^4 + \dots$$

$$B = \frac{1}{4} k^2 - \frac{1}{16} k^4 + \dots$$

$$C = \frac{1}{64} k^4 + \dots,$$

$$D = \dots,$$

$$E = \frac{5}{65536} k^8,$$

$$k^2 = e'^2 \sin^2 \beta_0$$

NOTATIONS:

a = Semi major axis for the spheroid chosen (WGS 72 is being adopted in the present computations)

b = Semi minor axis

e = first eccentricity = $2f - f^2$

e' = second eccentricity = $e^2 / (1 - e^2)$

f = flattening constant = $(a - b) / a$

ρ = radius of curvature in meridian = $\frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{3/2}}$

ν = radius of curvature in prime vertical = $\frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}}$

Software details:

MAIN PROGRAM (GEODESIC):

The source code has a main program 'GEODESIC' and two subroutines - 'INVERSE' and 'DIRECT'. The positions of two points - (LAT1, LON1); (LAT2, LON2) in degrees are read in the main program as input parameters while the spheroidal constants a and f are defined in the main program itself. The program uses common block for keeping values PI , spheroidal parameters like first and second eccentricity etc. The position coordinates are converted to radians and sent as input arguments to the subroutine INVERSE. The output DIST and AZ corresponding to the total distance and azimuth at the first coordinate return to the main program. Subsequent lines define the number of geodesic equations (NEQ), storage registers of arrays X, Y, Z in the form X(I), Y(I), Z(I) (I=1, NEQ), maximum number of steps (LAST), step size (ST) for integration. Now the intermediate points defining the geodesic at selected interval are computed using the routine DIRECT. For that the radius of curvature along meridian (ANEW) and prime vertical (V) at the intermediate point are computed. Subroutine DIRECT is called with arrays X, Y, Z and variables ANEW, ST, V, NEQ. The output is returned in the same array of X. Here X(1), X(2), X(3), X(4) contain the cumulative distance, intermediate coordinates and local azimuth of geodesic. All angles are converted into degrees while writing the results in the direct access disk file - 'INVERT:OUT'. This procedure is repeated till the cumulative distance becomes equal to DIST. While doing the

above, care is taken to compute the fractional step size (SF) at the last integration point so that the solution ends close to the terminal point.

SUBROUTINE INVERSE

This subroutine is called from the main program after initializing the spheroidal constants. LAT1, LON1, LAT2, LON2 are taken as input parameters from the calling program. The geodetic inverse problem is solved through an iterative procedure for computing the distance to the second point (DIST) and initial azimuth (ALP12). To start with, the difference between longitudes of the two points is assigned to L after initializing DIFF0 to zero. Starting with $L = \text{LAMBDA}$, it computes the difference (DIFF) between LAMBDA and L which is defined in the text. An improved LAMBDA is worked out and the computation is repeated. The new DIFF and the previous one (DIFF0) are compared with a preset tolerance limit. If the condition is not met with, then the value of DIFF is stored in DIFF0 and LAMBDA is updated. These steps are repeated till the desired refinement in LAMBDA is achieved. Then the azimuth (ALP12) and distance (DIST) are computed by using final LAMBDA. Results are sent to the main program as output to the calling program.

SUBROUTINE DIRECT

This subroutine is called from main program after setting the initial conditions for integration. The sequence of solving differential equations of geodesic is detailed in the flow chart.

The subroutine determines the geodesic coordinates and change of local azimuth using Runge-Kutta-Gills method and stores in the array X such that (X(I), I=1, NEQ) and returns to the calling program.

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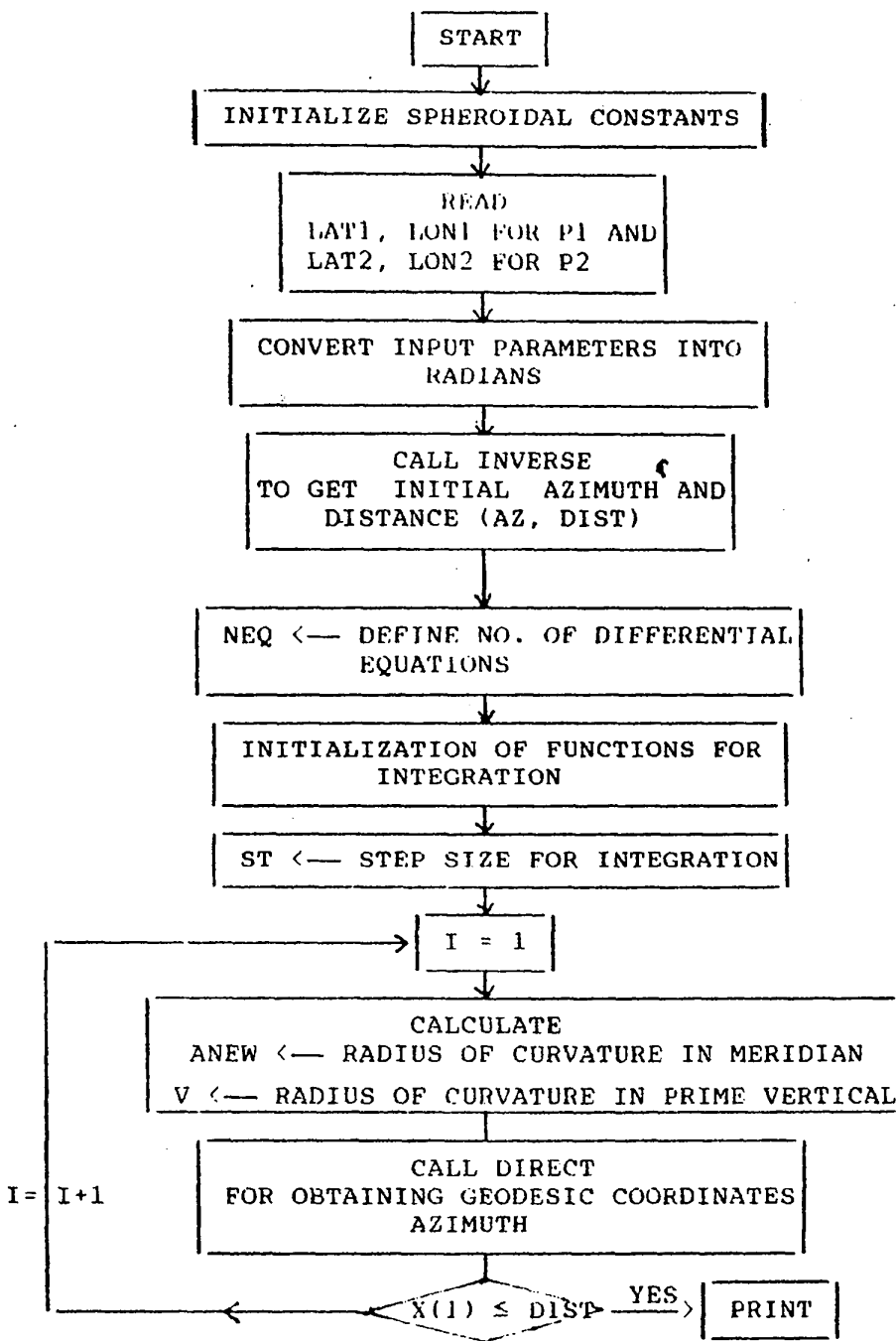
Legend to figures:

1. Schematic diagram of geodetic system.
2. Reduced sphere and ellipsoid.
3. Computed geodesic paths from Cape Leeuwin (Australia) towards the Indian coast.

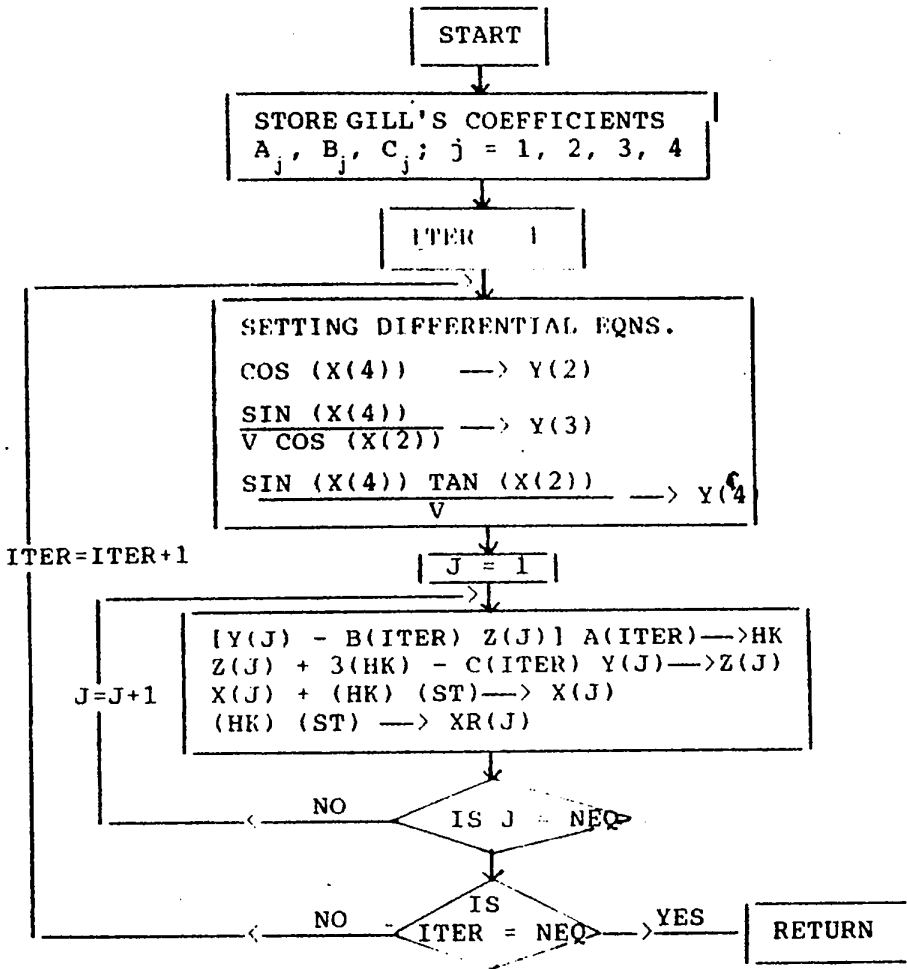
References:

1. Bomford, G. (1977) Geodesy, Oxford Univ. Press, 731 pp.
2. Ralston, A. (Editor), Mathematical Methods for Digital Computers, Vol.1, John Wiley & Sons, Inc. 1966. pp 1105-120.
3. Krakiwsky, E.J., and D.B. Thomson (1974) Geodetic position computations, Lecture Notes No. 39, Dept. of Survey Engg., Univ. of New Brunswick, Fredericton, 99 pp.

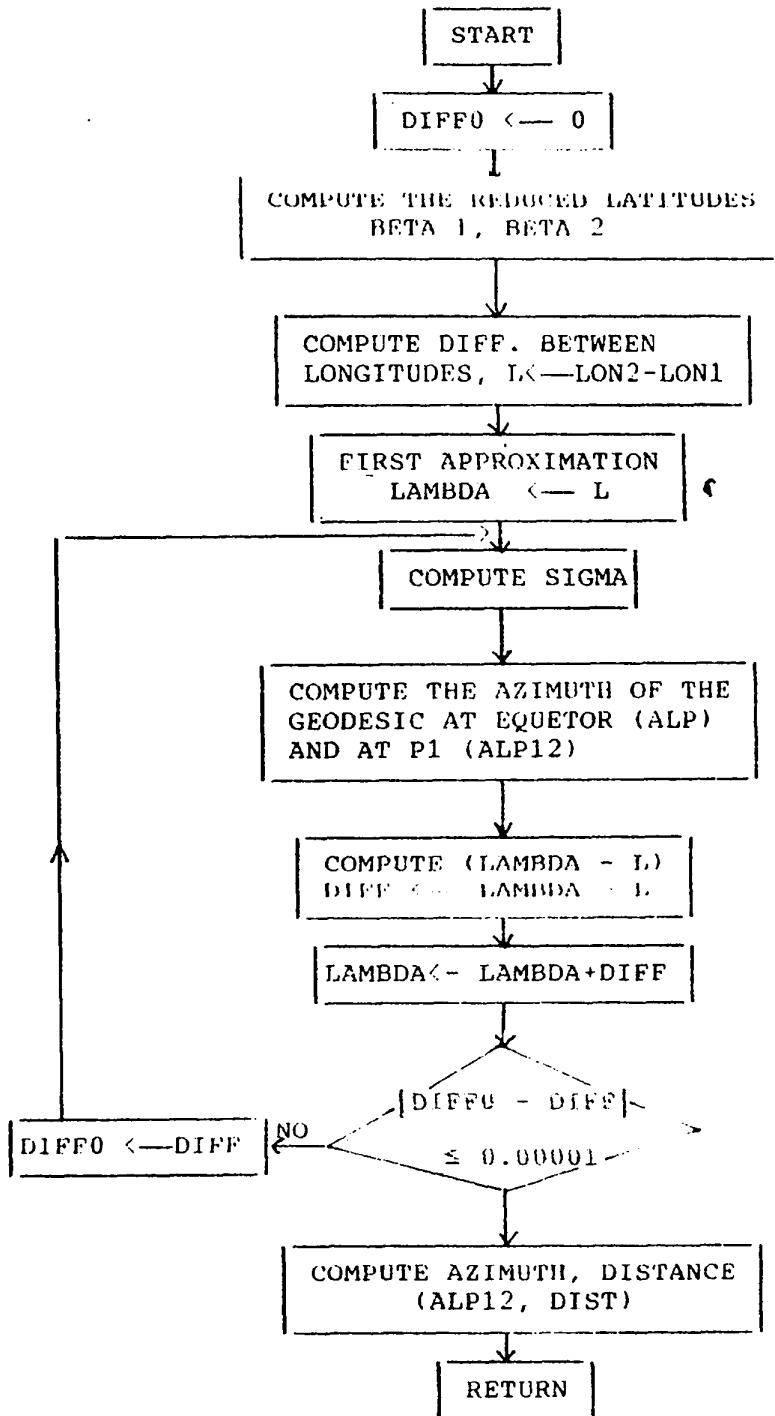
FLOW CHART OF THE MAIN PROGRAM



FLOW CHART OF SUBROUTINE 'DIRECT'



FLOW CHART OF SUBROUTINE 'INVERSE'



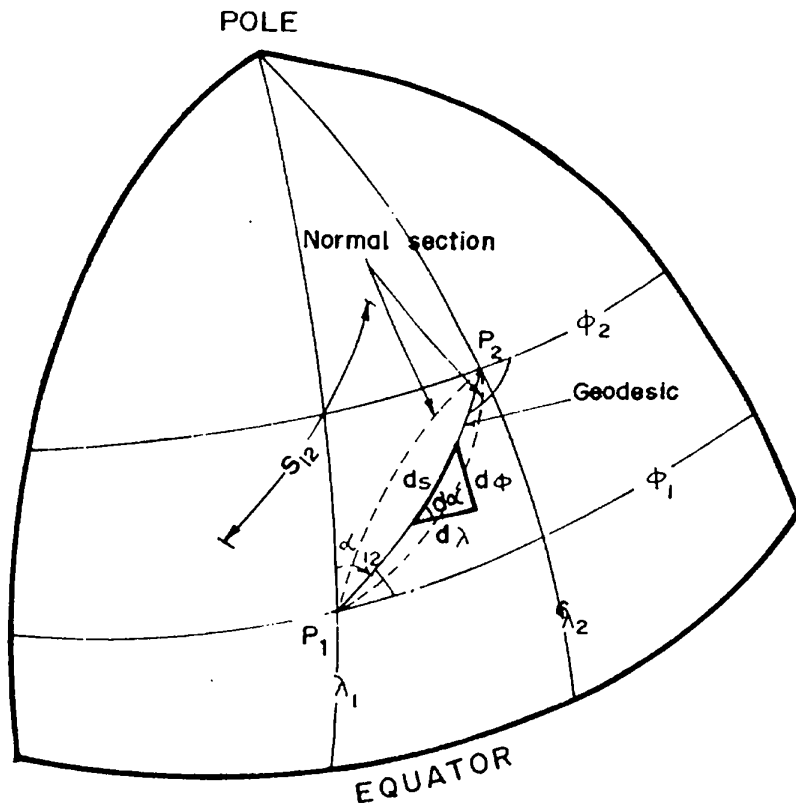


Fig. 1. Schematic diagram of Geodesic system

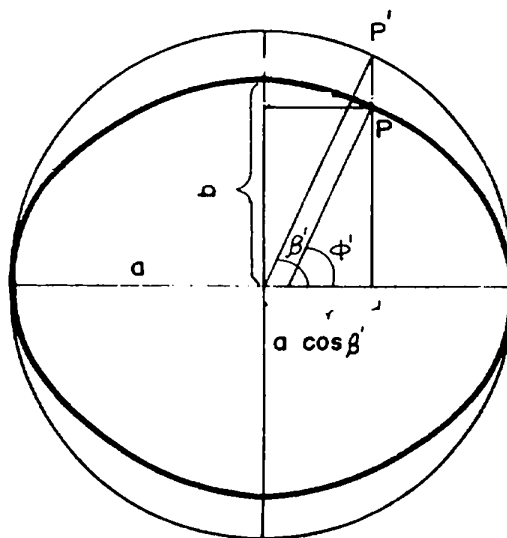


Fig. 2. Reduced Sphere and Ellipsoid

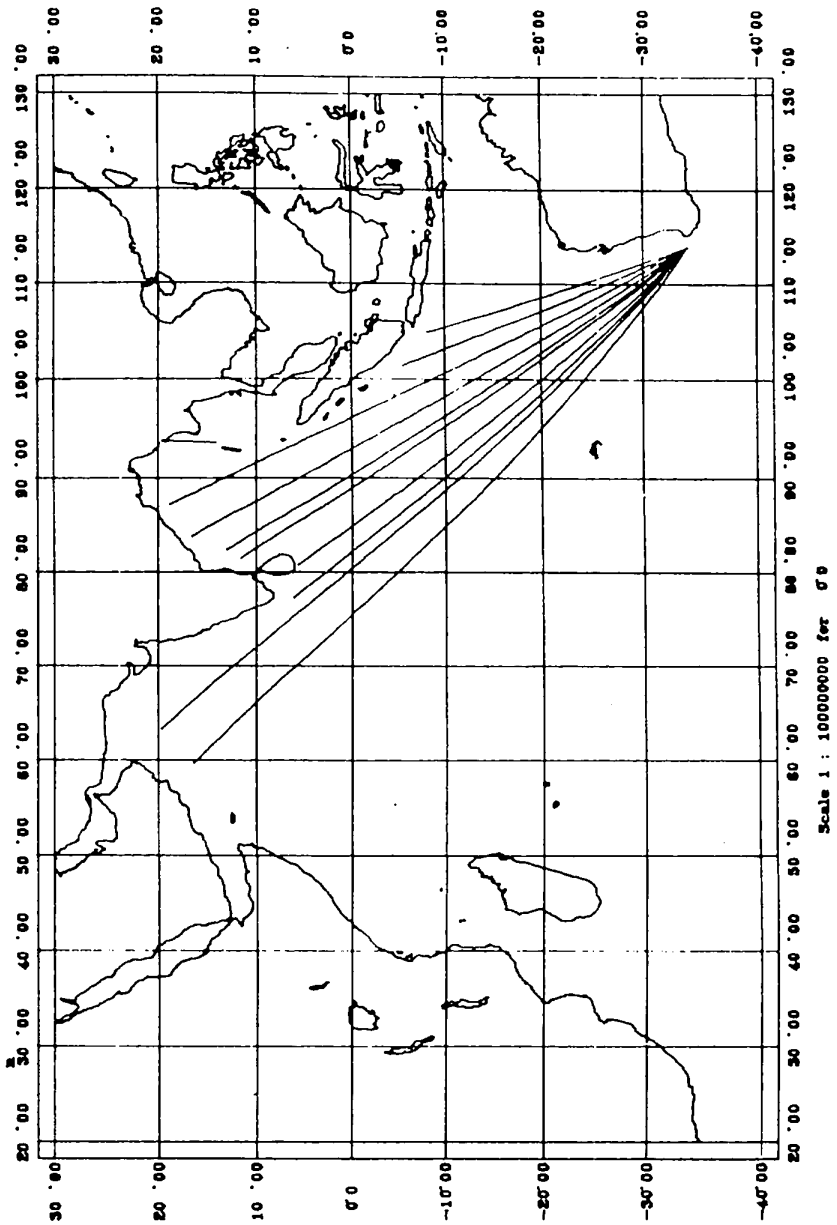


FIG. 5