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STATISTICAL ANALYSIS OF WAVE PARAMETERS - A CASE STUDY

S.S.BHAT. A
Lecturer, Dept.of
Applied Mechanics
Karnataka Regional
Engineering College
Srinivasnagar-574157

DR.I.V.NAYAK
Professor and Head
Dept.of Applied Mechanics
Karnataka Regional
Engineering College
Srinivasnagar-574157

DR.B.U.NAYAK
Head, Ocean Engg.Div.
National Inst.of
Oceanography
Dona Paula,
Goa-403004

ABSTRACT

Ocean waves of Bombay High region recorded during monsoon period of 1983 were analysed. The analysis was primarily intended to study the distributions of wave height and wave period of successive waves and some aspects of their joint distribution. The study indicates that the Rayleigh distribution for wave heights is valid well beyond the narrow band assumption. In the absence of a better theoretical distribution for wave periods, Longuet-Higgins theoretical model provides a reasonable fit to the data. Certain characteristics associated with the joint distribution proposed by Longuet-Higgins are not confirmed by the data. An attempt is made to find the difference in the wave characteristics if any, between growth and decay phases. But, with the limited data available no difference is observed.

INTRODUCTION

Wave analysis can be conducted in two methods: Statistical analysis and spectral analysis. But to bring out some useful statistical information regarding the basic characteristics of waves in a specific area, statistical analysis was carried out. Regarding the wave height distribution, significant amount of research work has been carried out. In comparison, the wave period distribution has received a scarce attention and the few cases studied, show considerable disagreement (Dattatri, 1973, Dattatri et al, 1979). Studies concerning the theoretical joint distribution of wave heights and periods are still fewer.

OCEAN WAVE DATA

The wave data was collected by Ocean Engineering Division, National Institute of Oceanography, Goa, in 1983, from Bombay High area, 180 km west of Bombay. Datawel wave rider accelerometer buoy was deployed to procure the data. The depth of water was 70 m. 54 records of July 1983, each being 20 minutes approximately, were analysed. All

computations were carried out at Ocean Engineering Division, National Institute of Oceanography, Goa, using Norsk-Data ND-100 computer.

PRESENTATION AND INTERPRETATION

The data is divided into two phases, growth and decay. The variation of H_{mean} shows the trend of data, whether growing or decaying (Fig.1). The characteristics of basic nature of wave parameters are compared. As the wind variability data was not available, the phasing is made by considering only the variation of H_{mean} following Rye (1974) and based on this definition for phases, the conclusions are made. Table I shows the different parameters used in the analysis.

Wave height distribution

Longuet-Higgins (1952) assuming that

- i) the instantaneous displacement of water surface is Gaussian,
 - ii) the wave spectrum contains narrow band of frequencies,
- proposed Rayleigh distribution for wave heights as,

$$p(H) = \begin{cases} H/\alpha^2 \exp(-H^2/2\alpha^2) & \text{for } H > 0, \quad \alpha > 0 \\ 0 & \text{for } H \leq 0 \end{cases} \quad \dots \quad (1)$$

where $\alpha = H_{\text{RMS}}/\sqrt{2}$, the distribution parameter.

In the present study, 54 individual records were compared with Rayleigh distribution and Chi-squared goodness of fit test was carried out. Only 20% of the total records analysed showed little deviation of χ^2 values at 0.05 level of significance. Average of all 54 records

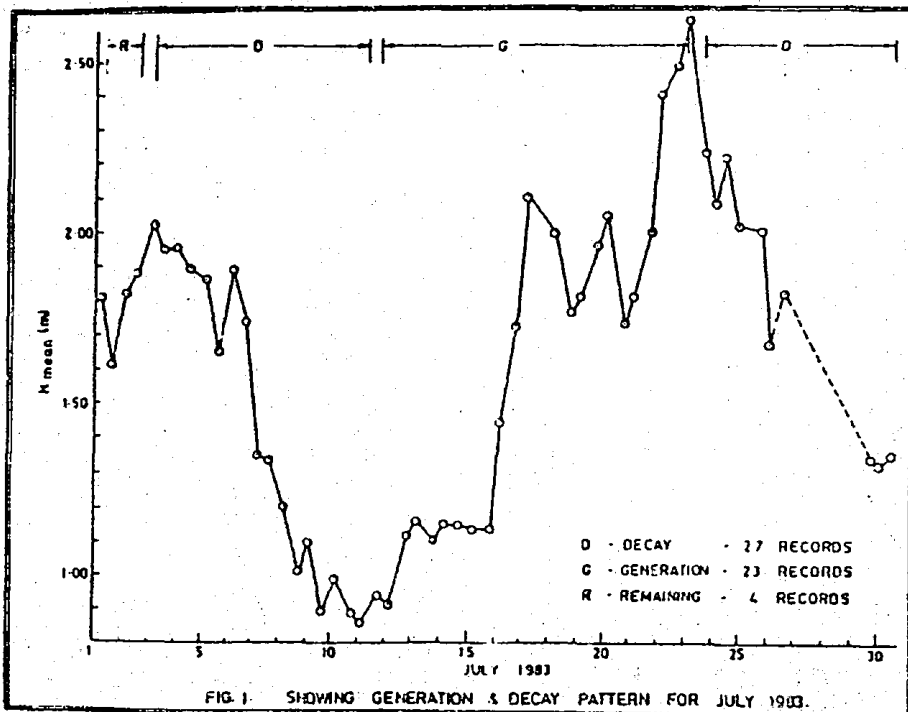


FIG. 1. SHOWING GENERATION & DECAY PATTERN FOR JULY 1983.

showed a good conformity with the theory (Fig.2). Table I shows the ϵ , spectral width parameter varies from 0.78 to 0.89, which necessarily belongs to wide band spectra. The ϵ spectral width parameter was evaluated as

$$\epsilon = [1 - (T_c/T_z)^2]^{1/2} \quad \dots \quad (2)$$

in which T_c is the mean crest period and T_z is the mean zero crossing period.

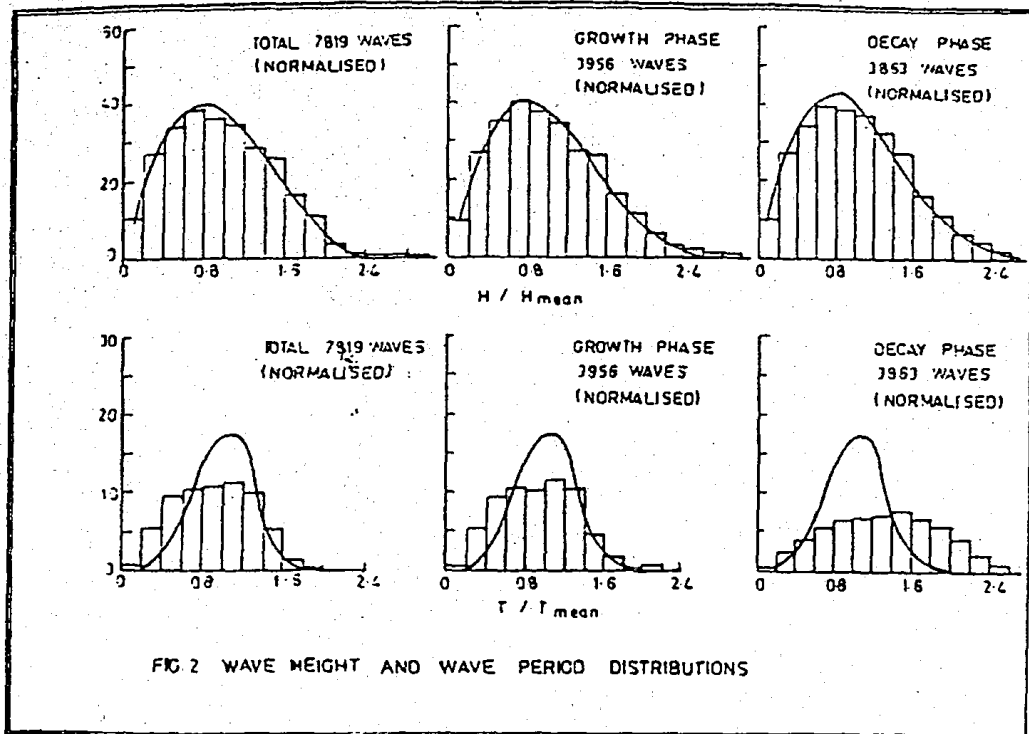


FIG. 2 WAVE HEIGHT AND WAVE PERIOD DISTRIBUTIONS

Even though the spectrum is sufficiently wide, the Rayleigh distribution approximation is still found to be good. Colonell and Perry (1970), Liu and Housley (1968), Dattatri et al, (1979), Ganesh Prasad (1985), Namboothiri (1985) arrived at similar conclusions. Average plot of decay and generation records also showed a good fit. Dattatri (1973) analysed the waves off Mangalore Harbour by considering seasonal effects, concluded that Rayleigh distribution holds good in any wave condition.

Wave period distribution

A theoretical model for wave period was first suggested by Bretschneider (1959) and later used by Chakrabarti and Snider (1974) with an assumption that square of the period follows the Rayleigh distribution. The distribution is given as

$$P(T)dT = 2.7(T/T_{mean})^4 (1/T) \exp[-0.675(T/T_{mean})^4] dT \quad \dots \quad (3)$$

where T is the wave period and T_{mean} is the mean wave period defined as the period around which most of the energy in the spectrum is concentrated.

TABLE I
PARAMETERS FOR OCEAN WAVE DATA ANALYSED

Rec.No.*		H _{mean} (m)	T _z (sec)	T _{mean} (sec)	
1	R	1.85	7.23	8.01	0.86
2	R	1.61	6.36	7.70	0.85
3	R	1.82	6.58	7.90	0.82
4	R	1.88	6.53	7.80	0.81
5	D	2.23	6.99	7.30	0.82
6	D	1.93	6.71	7.29	0.82
7	D	1.95	6.56	7.21	0.79
8	D	1.87	6.76	7.65	0.81
9	D	1.86	7.01	7.55	0.81
10	D	1.65	7.06	7.96	0.86
11	D	1.88	7.25	7.69	0.82
12	D	1.73	7.97	8.06	0.88
13	D	1.35	6.71	7.49	0.81
14	D	1.34	7.14	7.55	0.86
15	D	1.20	6.77	7.69	0.82
16	D	1.01	6.52	7.59	0.88
17	D	1.11	7.44	7.77	0.83
18	D	0.89	7.20	7.33	0.87
19	D	0.99	7.42	8.11	0.83
20	D	0.89	7.65	7.88	0.89
21	D	0.87	6.84	7.58	0.85
22	G	0.94	6.97	7.56	0.82
23	G	0.92	6.61	7.45	0.87
24	G	1.11	6.65	7.17	0.82
25	G	1.16	5.89	6.44	0.82
26	G	1.07	6.45	6.99	0.85
27	G	1.15	6.41	6.49	0.85
28	G	1.14	6.43	7.56	0.85
29	G	1.35	6.15	6.93	0.82
30	G	2.32	7.56	6.49	0.83
31	G	1.43	6.66	7.72	0.88
32	G	1.72	7.14	8.08	0.85
33	G	2.10	7.22	7.69	0.85
34	G	2.00	7.42	7.22	0.82
35	G	1.79	6.52	7.37	0.80
36	G	1.82	6.69	7.49	0.82
37	G	1.97	7.25	7.53	0.82
38	G	2.06	7.10	7.70	0.79
39	G	1.71	7.59	8.41	0.86
40	G	1.81	7.37	8.24	0.84
41	G	1.99	7.23	8.13	0.82
42	G	2.40	8.60	9.08	0.83
43	G	2.49	7.95	8.59	0.81
44	G	2.63	8.37	8.73	0.84
45	D	2.23	7.60	8.17	0.84
46	D	2.08	7.50	8.57	0.78
47	D	2.23	8.78	9.01	0.78
48	D	2.01	7.55	8.76	0.81
49	D	2.03	7.59	8.40	0.82
50	D	1.67	7.26	8.08	0.85
51	D	1.84	7.37	8.03	0.83
52	D	1.34	7.36	7.72	0.86
53	D	1.31	6.42	7.49	0.83
54	D	1.36	6.19	6.82	0.83

* G - Growth D - Decay R - Remaining (Fig.1).

Table I shows that the T_{mean} is greater than T_2 , hence to get a lesser value of $P(T)$, T_{mean} is used. The agreement of each wave record with the theory does not appear to be good at all. Chi-square goodness of fit test also shows the similar result. Average of all 54 records and both decay and growth phases also show disagreement (Fig.2). Dattatri et al (1979) concluded that Rayleigh distribution for square of the wave period does not hold good. But Chakrabarti and Snider (1974) had concluded that agreement of Rayleigh distribution with wave period appears to be better than that for wave height.

Longuet-Higgins (1975) proposed a symmetrical model for period distribution,

$$p(\eta) = \frac{1}{2}(1+\eta^2)^{-3/2} \quad \dots \quad (4)$$

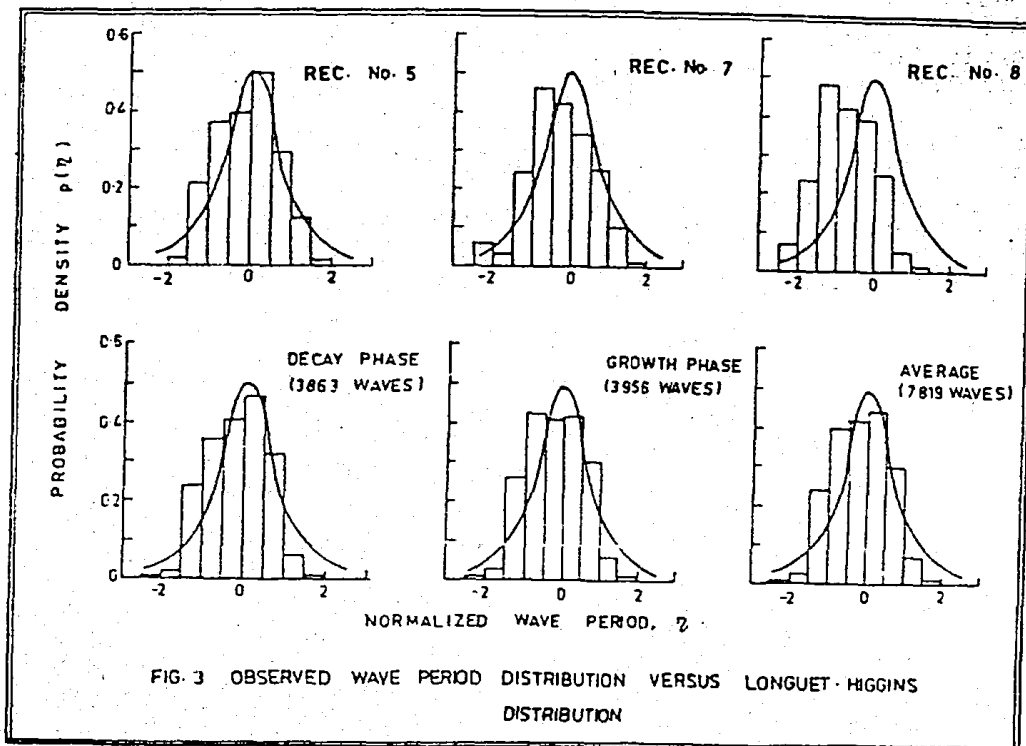
where $\eta = (T - T_{\text{mean}}) / \gamma T_{\text{mean}}$, normalised period

$$T_{\text{mean}} = m_0 / m_1$$

$$\gamma^2 = (m_2 m_0 - m_1^2) / m_1^2$$

in which m_0 , m_1 and m_2 are spectral moments about the origin of the energy spectrum.

The distribution of these wave periods for few representative samples are shown in Fig.3. The data is compared with Longuet-Higgins model, following Chakrabarti and Snider (1974) or Dattatri et al (1979). The observed data tend to be slightly skewed while the theoretical distribution is symmetrical. Considering the theoretical distribution



which assumes a narrow band spectrum, while the observed one, with ϵ ranging from 0.78 to 0.89 really belongs to wide band, the agreement can be termed as reasonable. Average of all samples which represents a smoothed version is also shown in Fig.3. Although a little skewness is observed, the Chi-square goodness of fit test at χ^2_{05} significance level, showed a reasonable agreement. Considering the average of decay and growth records, data of latter phase showed a better symmetrical distribution than the former. Chakrabarti and Snider (1974) and Dattatri et al (1979) found a rather higher skewness indicating a deficit of waves in higher period ranges. But, in the absence of a better theoretical model for wave period, Longuet-Higgins model can be assumed to be a reasonable fit.

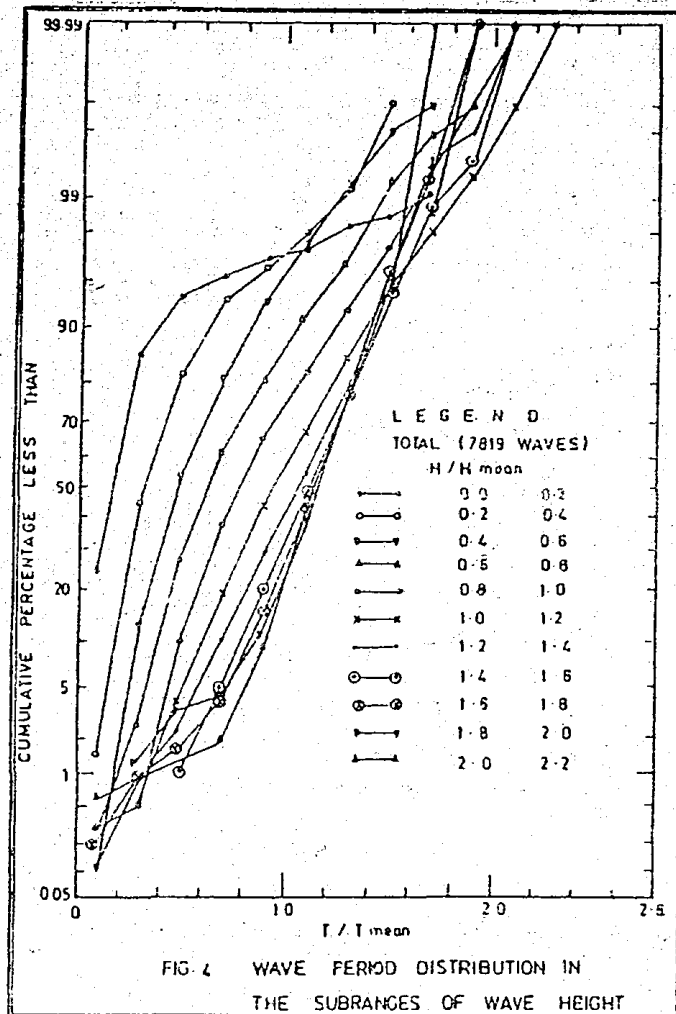
Joint distribution of wave height and wave period

The joint probability density of wave period and amplitude is given by Longuet-Higgins (1975) as

$$p(\epsilon, \eta) = [\epsilon^2 / (2\pi)^{1/2}] \exp [\epsilon^2 (1 + \eta^2) / 2] \dots (5)$$

where, $\epsilon^2 = a/\mu_0^{1/2}$ normalised wave amplitude
 $\mu_0 =$ zeroth moment of energy spectrum about the mean
 $a =$ amplitude.

The wave data corresponding to the different subranges of wave height are shown in Fig.4 for total 54 records, Fig.5 for generation phase and Fig.6 for decay phase, on a Gaussian scale. In all the cases, the conditional distributions are some what Gaussian but central quartile or median corresponding to each subranges of wave height goes on increasing particularly in the lower ranges. Only for the larger ranges of wave heights the median seems to remain constant which is indicated by merging of lines of distribution. Table II, III and IV show the Inter Quartile Range (IQR) for each subranges of wave height for total 54 records, growth and decay phases respectively. According to the theory, IQR for particular subrange goes on increasing as the wave height decrease. But this is true only in higher ranges of wave height. Similar tendency was observed by Cavanie et al (1976) and Dattatri et al (1979). No difference was noticed between the two phases.



CONCLUSIONS

The agreement of wave height distribution with the Rayleigh distribution is found to be very good in any wave condition even though the data did not satisfy the narrow band assumption. Rayleigh distribution for the square of wave period is found to be in total disagreement. However, fit of Longuet-Higgins (1975) model for wave period distribution can be termed as reasonable. Certain characteristics of theoretical joint distribution for the heights and periods of oceans waves as proposed by Longuet-Higgins (1975) is found to be not applicable to ocean wave data obtained on the west coast of India.

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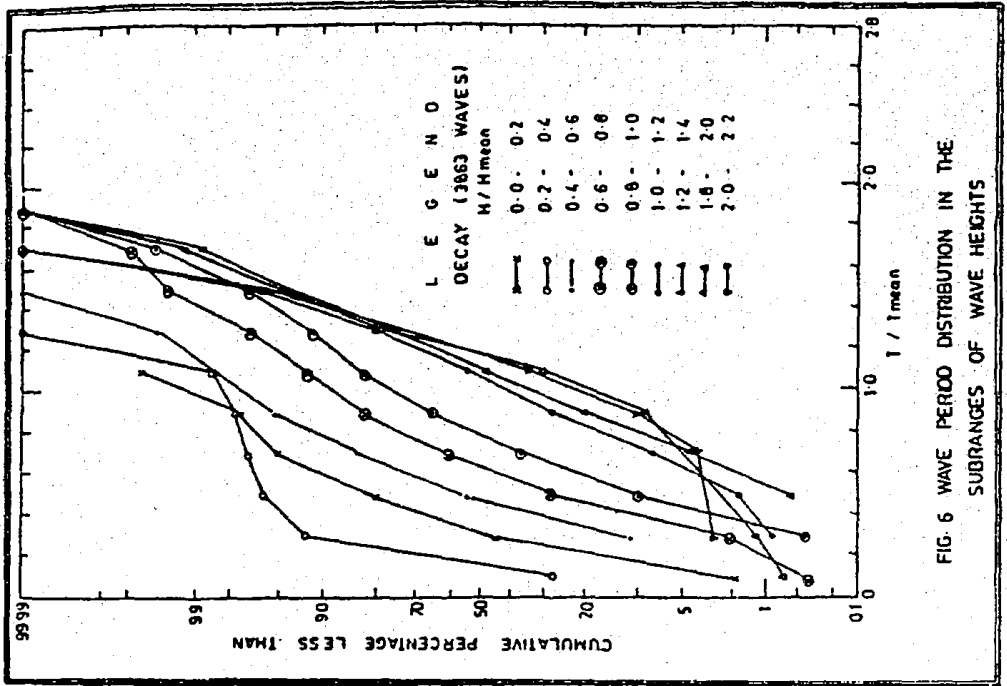


FIG. 6 WAVE PERIOD DISTRIBUTION IN THE SUBRANGES OF WAVE HEIGHTS

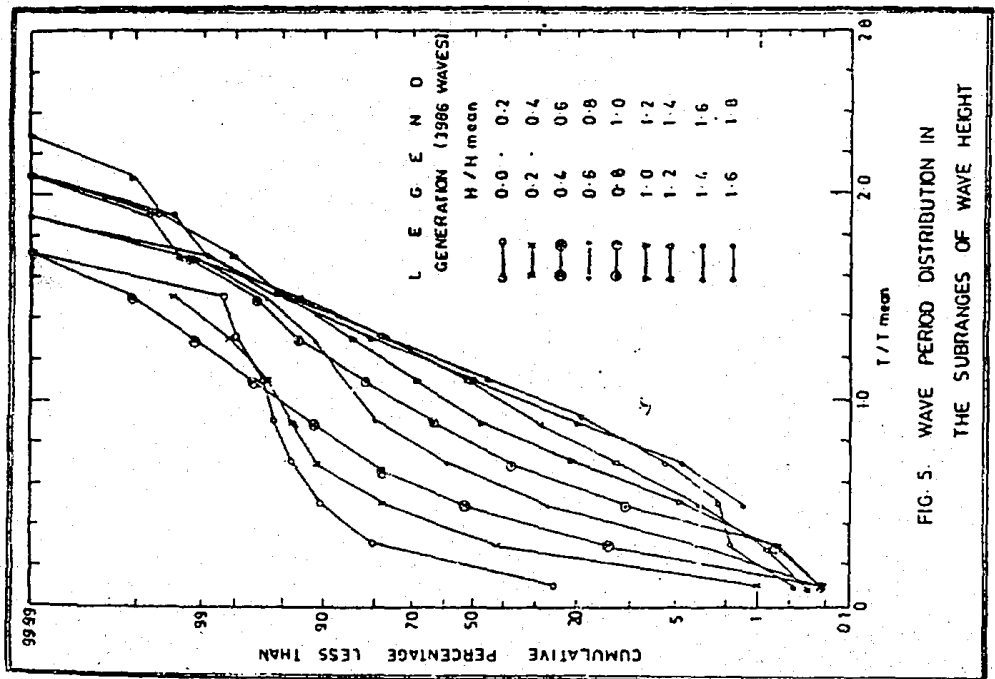


FIG. 5 WAVE PERIOD DISTRIBUTION IN THE SUBRANGES OF WAVE HEIGHT

TABLE II
 QUANTILES OF T/T_{mean} FOR SUBRANGES OF H/H_{mean}
 (Combined - 54 records)

Range H/H_{mean}	Number of waves	Quartiles T/T_{mean}			Inter Quartile Range
		Q_1	Q_2	Q_3	
0-0.2	275	-	0.276	0.355	-
0.2-0.4	752	0.306	0.403	0.514	0.208
0.4-0.6	958	0.406	0.517	0.715	0.345
0.6-0.8	1105	0.417	0.615	0.935	0.518
0.8-1.0	1061	0.703	0.809	1.111	0.408
1.0-1.2	1007	0.805	1.050	1.215	0.410
1.2-1.4	832	0.950	1.159	1.356	0.254
1.4-1.6	733	1.035	1.201	1.366	0.331
1.6-1.8	455	1.071	1.205	1.383	0.259
1.8-2.0	320	1.091	1.204	1.369	0.278
2.0-2.2	159	1.096	1.206	1.381	0.285
2.2-2.4	92	-	1.206	1.371	-
2.4-2.6	38	1.075	1.217	1.381	0.306
2.6-2.8	20	1.109	1.238	1.333	0.224
2.8-3.0	5	1.201	1.271	1.331	0.130
3.0-3.2	4	1.200	1.263	1.327	0.127
3.2-3.4	3	-	1.248	1.319	-
Total	7819				

TABLE III
 QUANTILES OF T/T_{mean} FOR SUBRANGES OF H/H_{mean}
 (Generation phase)

Range H/H_{mean}	Number of waves	Quartiles T/T_{mean}			Inter Quartile Range
		Q_1	Q_2	Q_3	
0-0.2	137	0.201	0.291	0.381	0.180
0.2-0.4	387	0.248	0.444	0.583	0.335
0.4-0.6	497	0.461	0.586	0.783	0.322
0.6-0.8	566	0.588	0.743	0.957	0.369
0.8-1.0	534	0.710	0.900	1.129	0.419
1.0-1.2	497	0.834	1.040	1.530	0.692
1.2-1.4	393	0.940	1.180	1.370	0.420
1.4-1.6	376	0.036	1.207	1.362	0.320
1.6-1.8	230	1.048	1.232	1.383	0.335
1.8-2.0	167	1.091	1.233	1.380	0.289
2.0-2.2	86	1.085	1.240	1.383	0.298
2.2-2.4	45	-	1.265	1.363	-
2.4-2.6	24	1.075	1.230	1.400	0.325
2.6-2.8	11	-	1.217	1.308	-
2.8-3.0	3	-	-	-	-
3.0-3.2	3	-	1.250	1.325	-
3.2-3.4	-	-	-	-	-
Total	3956				

TABLE IV
 QUANTILES OF T/T_{mean} FOR SUBRANGES OF H/H_{mean}
 (Decay phase)

Range H/H_{mean}	Number of waves	Quartile T/T_{mean}			Inter Quartile Range
		Q_1	Q_2	Q_3	
0-0.2	138	-	0.271	0.348	-
0.2-0.4	365	0.307	0.424	0.549	0.242
0.4-0.6	461	0.464	0.578	0.737	0.273
0.6-0.8	539	0.570	0.730	0.928	0.358
0.8-1.0	527	0.709	0.887	1.104	0.395
1.0-1.2	510	0.864	1.066	1.288	0.424
1.2-1.4	439	0.967	1.163	1.358	0.390
1.4-1.6	357	1.038	1.220	1.390	0.352
1.6-1.8	225	1.101	1.263	1.415	0.314
1.8-2.0	153	1.103	1.251	1.376	0.273
2.0-2.2	73	1.125	1.277	1.398	0.273
2.2-2.4	47	-	1.272	1.405	-
2.4-2.6	14	1.083	1.200	1.375	0.292
2.6-2.8	9	-	1.275	1.388	-
2.8-3.0	2	-	1.200	1.300	-
3.0-3.2	1	-	-	-	-
3.2-3.4	3	-	1.205	1.325	-
Total	3863				

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