

DETERMINATION OF WAVE DIRECTION FROM LINEAR AND POLYGONAL ARRAYS

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Abstract—Two key issues that have militated against popular adoption of methods based on phase/time/path difference concepts for the determination of wave direction from array measurements are addressed in this paper: the first issue being insufficient documentation of Borgman (1974) in case of linear arrays; and the second issue being the failure of Esteva (1976, 1977) to correctly determine wave directions over the design range 25 to 7 sec of his polygonal array. This paper presents requisite documentation for linear arrays, and shows that Esteva's array can, in fact, be used to advantage over the entire design range. For simulated sinusoidal wave trains, analysis of both linear and polygonal arrays consistently yielded directions accurately, within ± 1 degree in case of polygonal arrays. Results were unaffected when random noise was included for representing ocean waves more realistically.

INTRODUCTION

THE POPULARLY adopted approach for determining wave directions from array measurements is through the directional spectrum, in which, at each spectral frequency, energy (density) is evaluated as a function of direction, wherefrom the "mean" wave direction may be determined from the location of the energy peak, e.g. Borgman (1974) in case of polygonal arrays. This approach, pioneered by Barber (1963), assumes that the complicated pattern of waves results from the superposition of many long-crested sinusoidal waves; and is characterised by a widespread of energy in direction axis, the likely presence of subsidiary peaks and the occurrence of negative energy—all of which invariably are artifacts of the analysis. With the introduction of the concept of maximum likelihood estimators by Davis and Regier (1977), the problem of "spread" has been minimized.

The other approach for determining wave directions is through phase/time/path difference concepts, using cross-spectrum analysis. As in the "directional spectrum" approach, the sea surface is assumed to be the result of the superposition of a small number of narrow banded wave trains consisting of long-crested sinusoidal waves travelling in well defined directions—but with waves at a spectral frequency restricted to a single direction. In support of this approach, Esteva (1977) *inter alia* cites the work of Fujinawa (1974, 1975), "redundancy" and "expediency". Esteva (1976, 1977) and Carmel *et al.* (1985) determined "mean" or "representative" wave directions for different spectral bands using phase/time/path difference concepts.

Carmel *et al.* (1985), employing coherence function to ascertain validity of inherent assumptions, used two gauges, 10.8 m apart in 6.3 m of water to determine wave directions very accurately for waves in the range 15 to 4 sec. Galvin (1985) pointed out that the gross annual transport computed by Carmel *et al.* (1985) was in agreement with his empirical relation based on average breaker height.

At the Pt Mugu site of the polygonal array of Esteva (1976, 1977) swell of 16 sec and 8 sec are observed. For both observed and simulated data, using "consistency" within redundant values of wave direction as a criterion, Esteva reported, success in determining wave directions for swell of 16 sec, and failure for swell of 8 sec, although his array had been designed for waves of 25–7 sec.

This paper presents a unified treatment for the determination of wave directions from the time series of wave elevation simultaneously measured at several gauges arranged in linear or polygonal arrays. For linear arrays, the analysis is performed in units of gauge pairs in lines with Borgman (1974) and Carmel *et al.* (1985). And for polygonal arrays, the analysis is performed in units of gauge triads in lines with Esteva (1976, 1977). Results of gauge pair and gauge triad analyses of four cases each, of wave trains of 16 sec and 8 sec swells, simulated at Esteva's polygonal array, are presented. For *all* the simulated wave trains, both the gauge pair and gauge triad analyses consistently yielded directions accurately (within ± 1 degree for gauge triads). This was achieved by employing a simple criterion first enunciated by Barber and Doyle (1956). The use of this criterion for designing arrays is demonstrated.

PHASE DIFFERENCE

Using cross-spectrum analysis the phase difference ϕ_{ij} between the arrival of a wave crest at two gauges i and j , at a spectral frequency f may be obtained uniquely in the range $(-\pi, \pi)$ from the following equation by considering the signs in the numerator and denominator:

$$\phi_{ij}(f) = \arctan [Q(f)/C(f)] \quad (1)$$

where $Q(f)$ and $C(f)$ are the quadrature and cospectrum respectively.

If the coherence at this frequency is statistically significant, the phase difference ϕ_{ij} determined by the above equation will correctly represent the physical situation provided that the criterion of Barber and Doyle (1956)

$$D < \lambda/2$$

is satisfied, where D is the distance between the two gauges and λ is the wavelength corresponding to the spectral frequency f . When the above criterion is satisfied, the phase difference in the physical situation is restricted to the interval $(-\pi, \pi)$ so that ϕ_{ij} is unambiguously determined by Equation (1).

LINEAR ARRAY

Consider a sinusoidal long crested wave of spectral frequency f approaching gauges 1 and 2, separated by distance D , at an angle α (see Fig. 1). Further suppose that the criterion $D < \lambda/2$, is satisfied. The time difference τ between the arrival of a wave crest at the two gauges is given by

$$\tau = \phi \cdot T / (2\pi)$$

where ϕ is the phase difference computed using Equation (1). The wave direction α , may then be determined from the equation

$$D \cos \alpha = c\tau \quad (2)$$

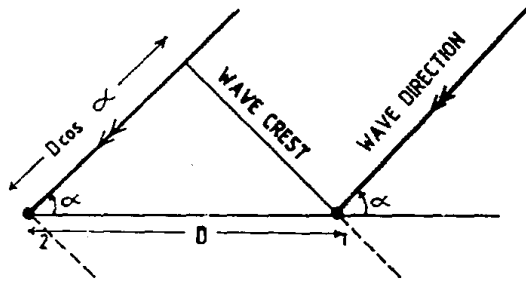


FIG. 1. Determination of path difference between the arrival of a wave crest at two gauges in case of linear arrays.

where the wave celerity c is a function of f alone. To evaluate c , the wave number κ , corresponding to frequency f is first determined from the following well known formula of linear wave theory, by the method of iteration:

$$(2\pi f)^2 = \kappa g \tanh \kappa d \tag{3}$$

where d is the mean water depth at the gauge sites. The wave length λ ($\lambda = 2\pi/\kappa$) can then be determined and the celerity, $c(= \lambda/T = \lambda f)$ evaluated.

We note that if ϕ is positive, α is acute and if ϕ is negative, α is obtuse. But we are unable to distinguish waves arriving at the gauges at the same angle but from opposite sides, i.e. at angles α and $2\pi - \alpha$ (see Fig. 1). However, oftentimes it may be possible to assume that waves come from just one side, the seaward side, e.g. swell at a long coastline.

At this stage one may conclude that should our interest lie in determining directions for waves of period larger than, say, 5 sec, then a single pair of gauges installed at 10-m depth at a distance D apart, $D < 18.3 \text{ m} = \lambda/2$, should suffice. But because of another important consideration, namely, accuracy and resolution, in practice, several gauges are installed in a linear array with gauge pair separations generally $D, 2D, 3D, \dots$, etc., the larger separations determining directions of waves of higher period (see Fig. 2).

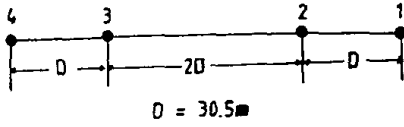
POLYGONAL ARRAY

More often than not it is likely that waves approach from all sides, for example in cases of sea and/or swell at locations far from the shore or near promontories. But as noted earlier, linear arrays are not able to distinguish between waves arriving at the same angle but from opposite sides (this defect is sometimes referred to as an ambiguity within a mirror-symmetry). For such cases therefore linear arrays fail in determining directions correctly. We shall show how polygonal arrays may be used for precisely such cases.

Let $(x_i, y_i), i = 1, 2, 3$ be the positions of three gauges, referred to convenient axes. Let α be the angle made by the wave with the positive direction of the x -axis, the angle being measured anticlockwise (see Fig. 3). Then at each spectral frequency the wave direction α is uniquely determined from the following equation by considering the signs of the numerator and denominator (see Esteva, 1977)

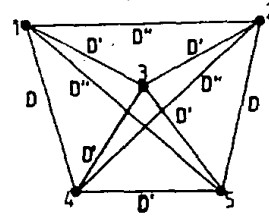
LINEAR ARRAYS

EX 1 : PAWKA, 1974



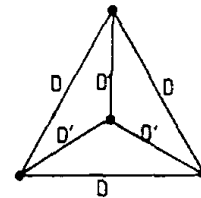
POLYGONAL ARRAYS

EX 1 : ESTEVA, 1977



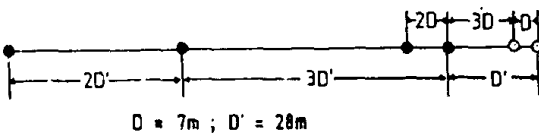
$D = 30.08$ $D' = 25.60$ $D'' = 48.77$
(metres)

EX 2 : BORGMAN, 1974



$D = 251.4$; $D' = 124.4$

EX 2 : HASSELMANN et al, 1973



$D = 7m$; $D' = 28m$

FIG. 2. Some examples of linear and polygonal arrays.

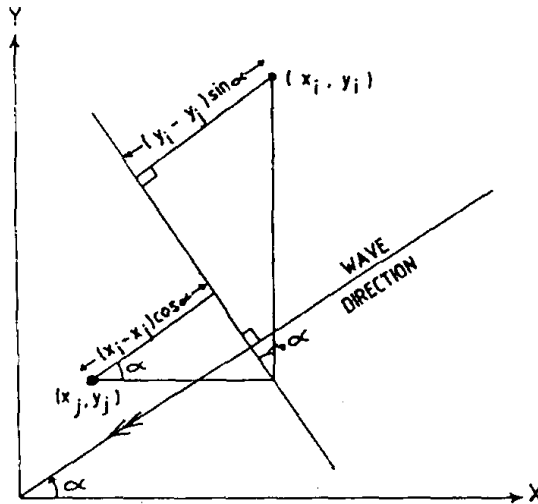


FIG. 3. Determination of path difference between the arrival of a wave crest at two points (x_j, y_j) and (x_i, y_i) in case of polygonal arrays.

$$\alpha = \arctan \frac{[(x_1 - x_2)\phi_{13} - (x_1 - x_3)\phi_{12}]/\text{sgn } P}{[(y_1 - y_3)\phi_{12} - (y_1 - y_2)\phi_{13}]/\text{sgn } P} \quad (4)$$

where, ϕ_{12} and ϕ_{13} are the phase differences between gauge pairs 1-2 and 1-3, respectively,

$$P = \kappa[(x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)] \quad (5)$$

does not vanish for nonlinear arrays, κ is the wave number, and by definition, $\text{sgn } P = 1$, for $P > 0$ and $\text{sgn } P = -1$, for $P < 0$.

Clearly the direction α determined above will be correct only if both D_{12} and D_{13} are less than half the wavelength at the spectral frequency under consideration, where D_{12} , D_{13} are the distances between gauges 1-2 and 1-3, respectively. We note that three gauges are sufficient to determine wave direction, the distances between the gauges placing a lower limit on the wave period at which they can be unambiguously determined.

The different combinations of gauge triads may be grouped into three distinct classes for the purpose of determination of directions at a specified spectral frequency.

(1) Class "YES", composed of gauge triads all of whose gauge separations are within half a wavelength at the specified frequency, thereby always giving correct directions.

(2) Class "IF", composed of those gauge triads that contain a single gauge separation, i.e. one arm of the triangle, which exceeds half the wavelength at the specified frequency, thereby giving correct directions if we do not use that particular arm in the computations of phase differences. This can easily be done.

(3) Class "NEVER", composed of those gauge triads that contain at least two arms of the triangle that exceed half the wavelength. In this case directions determined can be correct only by chance!

Since as seen above, for a triad belonging to class "IF", the choice of gauge pairs used for determining phase differences is crucial to the analysis, we use the following convention: A triad is considered as an ordered sequence of gauges i - j - k for which we compute phase differences ϕ_{ij} , ϕ_{ik} .

DESIGN AND INTERPRETATION

Figure 4 is a plot between the wave period T and the maximum gauge separation permissible, i.e. half a wavelength which as we have already seen ensures that directions are determined correctly. The plot is valid for depths $d = 10, 9.14$ m.

Figure 5 shows time difference τ plotted against wave direction α for wave periods 4, 8, 12 and 16 sec at three gauge separations $D = 20, 50$ and 100 m. The dashed curves signify that the criterion $D < \lambda/2$ is not valid as the time difference τ is not less than $T/2$. We note that in case of period $T = 12$, directional information is correct for gauge separations 20 and 50, and a further perusal of the same figure tells us that for gauge separation $D = 50$, angles are better resolved, specially for $\alpha < 40^\circ$. In practice, for small α , the resolution will further suffer because time difference obtained from directional analysis will be accurate at best to within $\pm h$, the sampling interval of the time series elevation, the least value of h generally used being 0.25 sec. Besides their ambiguity within a mirror symmetry, this is the other serious defect of linear arrays, as they can be used to resolve directions well only when waves approach nearly normal to the array.

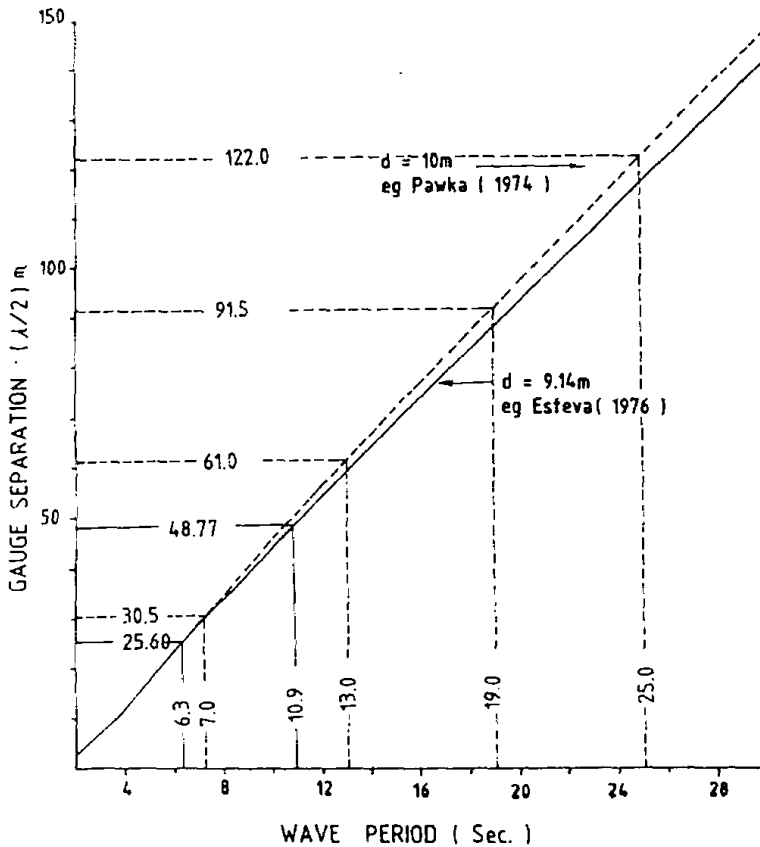


FIG. 4. Relationship between wavelength and wave period for water depths 10, 9.14 m.

To illustrate the use of Fig. 4 we shall examine the linear array of Pawka (1974) installed at a mean water depth of 10 m. The ${}^4C_2 = 6$ possible combinations of gauge pairs along with their gauge separations and the wave periods above which the pairs will correctly determine wave directions, are given in Table 1. For gauge identification please see Fig. 2. We note from Table 1 that two pairs 1-2 and 3-4 should be used for determining directions correctly and with the best possible resolution for wave periods between 7 and 13 sec. Gauge pair 2-3 should be used for periods in the range 13-19 sec, gauge pairs 1-3 and 2-4 for periods in the range 19-25 sec, and for periods larger than 25 sec one has no choice but to use the gauge pair 1-4.

We shall now study the characteristics of the polygonal array of Esteva (1976) installed at a mean water depth of 9.14 m. From gauge separations given in Fig. 2 and our classification of gauge triads, we surmise that we can only use the four ordered triads 3-1-2, 3-1-4, 3-4-5, and 3-5-2 for determining wave directions correctly in case of wave periods between 6.3 and 7.1 sec. The ordered triads 4-1-5 and 5-4-2 may be used for periods between 7.1 and 10.9 sec for best possible resolution. And for periods larger than 10.9 sec, the two ordered triads 2-1-4 and 1-2-5 will ensure the best possible resolution. We also note that the array provides for redundant analyses for all wave periods larger than 6.3 sec.

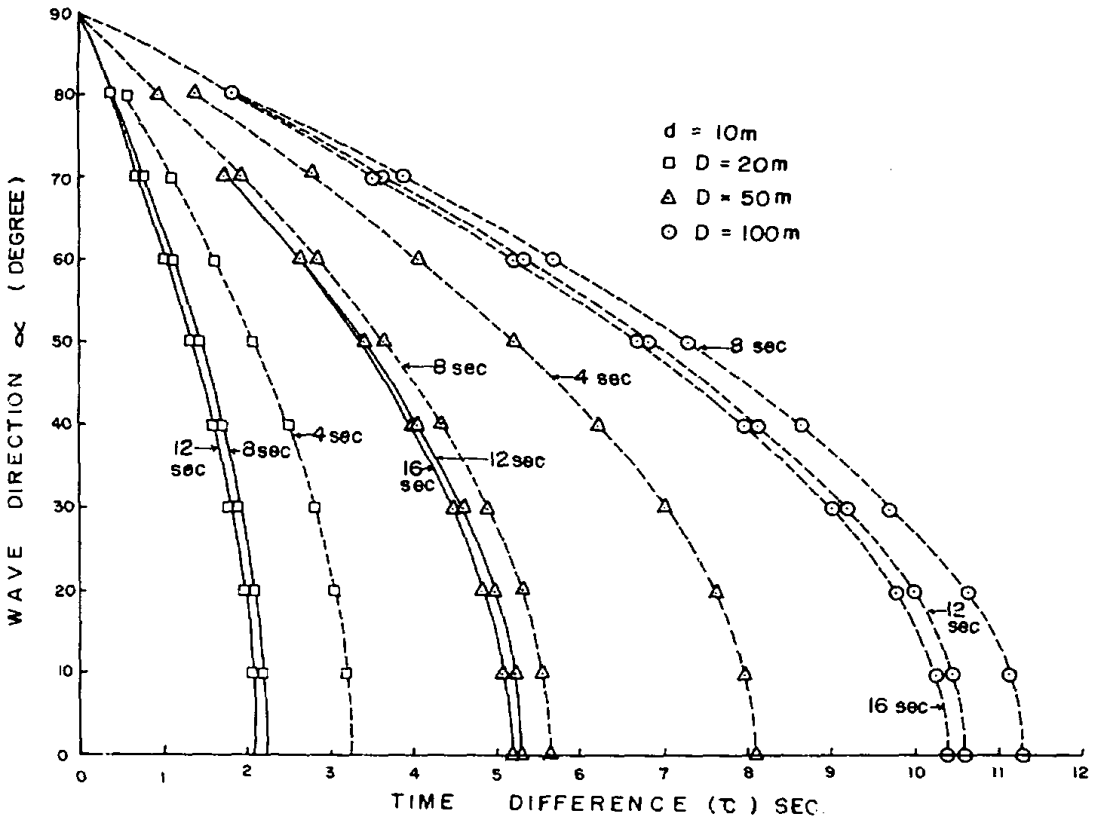


Fig. 5. Relationship between time difference between arrival of a wave crest at two gauges and wave direction, for gauge pair separations 20, 50 and 100 m. A mean water depth of 10 m is assumed.

TABLE 1. CHARACTERISTICS OF THE LINEAR ARRAY OF PAWKA (1974)

Gauge pairs (i-j)	Separation (m)	Limiting period (sec)	Best resolution (sec)
1-2, 3-4	30.5	7	7-13
2-3	61.0	13	13-19
1-3, 2-4	91.5	19	19-25
1-4	122.0	25	>25

In Fig. 7, we present a design for a polygonal array to be placed at 10 m depth, for handling period ranges 7-13, 13-19 sec and periods larger than 19 sec. The design made on lines of the linear array of Pawka (1974), may not be practical, but should, nevertheless, serve the purpose of illustration of design capability of Fig. 4.

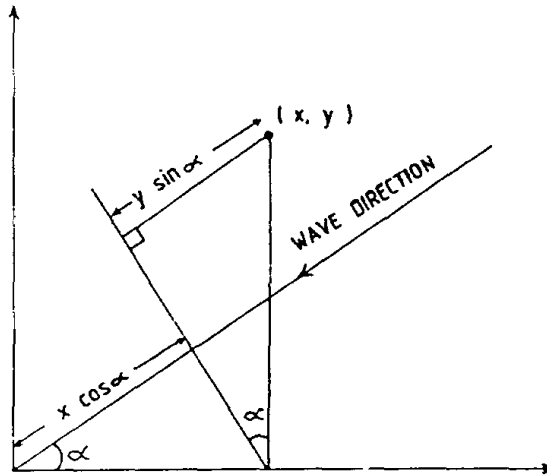


FIG. 6. Determination of path difference between arrival of a wave crest at some point (x,y) and at the origin.

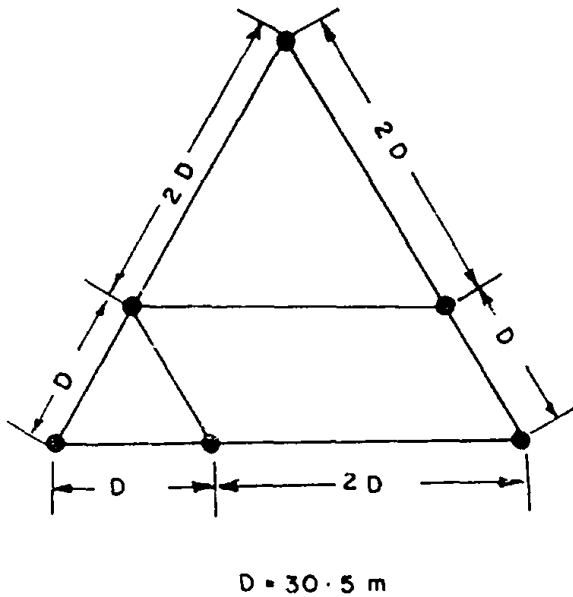


FIG. 7. Design for a polygonal array at 10 m depth, having good resolution for the period range 25–7 sec.

SIMULATION STUDIES

Wave simulation

Let (x,y) be the position of a gauge referred to convenient axes, and α , the wave direction reckoned positive anticlockwise (see Fig. 6). Then the wave elevation, η at the gauge as a function of time, t is given by

$$\eta = A \text{Cos} [2\pi ft + \theta + \kappa(x \text{cos}\alpha + y \text{sin}\alpha)] \tag{6}$$

where f is the wave frequency, κ is its corresponding wave number and θ is the initial phase at the origin and A is the amplitude. The above equation was used to simulate several cases of sinusoidal wave fields at the five gauges of Esteva (1976, 1977) (see Fig. 2).

Each simulated wave field was constructed from three sinusoids, each sinusoid being assigned a specific direction, nearly equal amplitude and frequency 0.134/1024 (≈ 0.00012) Hz removed from the closest spectral frequency in lines with Esteva, but having arbitrary phase at the origin. Sampling interval h and number of points N in the time series were taken as 1 sec and 1024 respectively (this is analogous to $h = 0.25$ and $N = 4096$ used by Esteva!). The characteristics of eight cases of wave fields generated are given in Table 2. The first four may be briefly described as 16 sec swell and the remaining four cases as 8 sec swell. In cases (1), (2), (5) and (6) frequencies of the three simulated sinusoids are 3/1024 Hz apart. And in cases (3), (4), (7) and (8)

TABLE 2. CHARACTERISTICS OF SIMULATED WAVE TRAINS

	Closest spectral		Simulated wave train			
	Line (k)	Period (sec)	Amplitude (cm)	Period* (sec)	Phase† (deg)	Direction (deg)
Case (1)	61	16.79	11.43	16.75	15	21
	64	16.00	15.24	15.97	25	21
	67	15.28	11.43	15.28	20	21
Case (2)	61	16.79	11.43	16.75	15	21
	64	16.00	15.24	15.97	25	15
	67	15.28	11.43	15.28	20	-21
Case (3)	62	16.79	11.43	16.75	15	21
	64	16.00	15.24	15.97	25	21
	65	15.28	11.43	15.28	20	21
Case (4)	62	16.79	11.43	16.75	15	21
	64	16.00	15.24	15.97	25	15
	65	15.28	11.43	15.28	20	-21
Case (5)	125	8.19	11.43	8.18	10	-60
	128	8.00	15.24	7.99	20	-60
	131	7.82	11.43	7.81	-15	-60
Case (6)	125	8.19	11.43	8.18	10	-60
	128	8.00	15.24	7.99	20	-40
	131	7.82	11.43	7.81	-15	60
Case (7)	126	8.19	11.43	8.18	10	-60
	128	8.00	15.24	7.99	20	-60
	129	7.82	11.43	7.81	-15	-60
Case (8)	126	8.19	11.43	8.18	10	-60
	128	8.00	15.24	7.99	20	-40
	129	7.82	11.43	7.81	-15	60

*Indicates approx. period.

†Indicates initial phase at the origin.

the first and second sinusoid are 2/1024 Hz apart, while the second and third are 1/1024 Hz apart. In cases (1), (3), (5) and (7) the sinusoids are assigned a common direction while different directions are assigned in the other cases. The theoretical basis for the simulation technique has been given by Esteva (1977).

We can look upon gauge pairs 2-1 and 5-4 as two separate gauge pairs in addition to ${}^5C_3 = 10$ possible combinations of gauge triads, viz. 1-2-3, 1-2-4, 1-2-5, 1-3-4, 1-3-5, 3-4-5, 3-2-5, 1-4-5, 2-4-5, and 3-2-4. Hanning window in time domain was used for reducing spillover of energy in accord with Esteva (1977). However spectral densities (amplitude squared) reported in this paper have been scaled up by a factor of 1/0.375 to compensate for loss of energy due to the window used.

Analysis of 16-sec swell

For cases (1)-(4), i.e. 16-sec swell, we note from Fig. 2 that the highest gauge separation in the array of gauges is 48.77 m which is well within half a wavelength ($\lambda/2 = 68$ m) at this wave period (see Fig. 4). We therefore note that the two gauge pairs and all ten gauge triads belong to the "YES" class. Table 3 presents the results of the analyses of gauge pairs and gauge triads for case (2). We see that the directions obtained are close to the assigned direction, except perhaps in energy troughs. This is true for cases (1) and (3) as well, and not exactly true for case (4), where the frequency separation of simulated sinusoids is less than 3/1024 Hz and they are assigned widely different directions. These results are in agreement with Esteva (1976, 1977).

TABLE 3. RESULTS OF LINEAR AND POLYGONAL ANALYSES FOR CASE (2): 16 sec SWELL FROM 21°, 15°, -21°

Line (k)	Spectral		Computed directions for triads <i>ijk</i> and pairs <i>ij</i>											
	Period (sec)	Density (cm ²)	123	214	125	134	135	145	234	235	245	345	21	54
58	17.66	0.00	19	19	19	19	19	19	19	19	19	19	0	0
59	17.36	0.03	20	20	20	20	20	20	20	20	20	20	11	13
60	17.07	14.22	21	21	21	21	21	21	21	21	21	21	21	17
61*	16.79	83.96	21	21	21	21	21	21	21	21	21	21	21	21
62	16.52	33.97	21	21	21	21	21	21	21	21	21	21	23	23
63	16.25	22.53	15	15	15	15	15	15	15	15	15	15	10	10
64*	16.00	151.71	15	15	15	15	15	15	15	15	15	15	15	15
65	15.75	57.83	14	14	14	14	14	14	14	14	14	14	14	18
66	15.52	11.59	-24	-24	-24	-24	-24	-24	-24	-24	-24	-24	-24	18
67*	15.28	86.59	-21	-21	-21	-21	-21	-21	-21	-21	-21	-21	-21	21
68	15.06	31.07	-21	-21	-21	-21	-21	-21	-21	-21	-21	-21	-21	23
69	14.84	0.08	-19	-19	-19	-19	-19	-19	-19	-19	-19	-19	-19	25
70	14.63	0.00	-16	-16	-16	-16	-17	-16	-16	-16	-16	-16	-17	27

*Indicates spectral line closest to the frequency of the simulated wave.

Analysis of 8-sec swell

For cases (5)-(8), i.e. 8-sec swell, the position is complex. For this period the gauge separations for three pairs, viz. 1-2, 1-5 and 2-4 each of which is 48.77 m exceeds half the wavelength ($\lambda/2 = 34$ m) at this period (see Figs 2, 4). Therefore in the case of gauge pairs, the pair 2-1 may not yield correct directions whereas the pair 5-4 always

will. In the case of gauge triads, we note that three triads, viz. 3-1-4, 3-4-5 and 3-2-5 belong to the class "YES", four triads, viz. 3-1-2, 4-1-5, 5-2-4 and 3-2-4 belong to the class "IF" and two triads, viz. 1-2-5 and 1-2-4 belong to the class "NEVER". Tables 4-7 give the results for the gauge pair 5-4 and for gauge triads belonging to class "YES" and "IF". We note that the gauge triads faithfully reproduce the assigned direction for cases (5)-(7), and so does the gauge pair 5-4, which in addition demonstrates the ambiguity of direction within a mirror symmetry for linear arrays. Case (6) was re-run

TABLE 4. RESULTS OF LINEAR AND POLYGONAL ANALYSES FOR CASE (5): 8-SEC SWELL FROM -60°, -60°, -60°

Spectral			Computed directions for triads <i>ijk</i> and pairs <i>ij</i>											
Line (k)	Period (sec)	Density (cm ²)	123	214	125	134	315	415	324	235	524	345	21	54
			Degrees											
122	8.39	0.00				-60	-60	-60	-60	-60	-60	-60		58
123	8.33	0.03				-60	-60	-60	-60	-60	-60	-60		59
124	8.26	14.22	I	I	I	-60	-60	-60	-60	-60	-60	-60	I	59
125*	8.19	83.95	n	n	n	-60	-60	-60	-60	-60	-60	-60	n	60
126	8.13	33.94	v	v	v	-60	-60	-60	-60	-60	-60	-60	v	60
127	8.06	22.52	a	a	a	-60	-60	-60	-60	-60	-60	-60	a	60
128*	8.00	151.89	l	l	l	-60	-60	-60	-60	-60	-60	-60	l	60
129	7.94	57.44	i	i	i	-60	-60	-60	-60	-60	-60	-60	i	60
130	7.88	12.03	d	d	d	-60	-60	-60	-60	-60	-60	-60	d	60
131*	7.82	86.34				-60	-60	-60	-60	-60	-60	-60		60
132	7.76	31.13				-60	-60	-60	-60	-60	-60	-60		61
133	7.70	0.08				-60	-60	-60	-60	-60	-60	-60		61
134	7.64	0.00				-60	-60	-60	-60	-60	-60	-60		61

*Indicates spectral line closest to the frequency of the simulated wave.

TABLE 5. RESULTS OF LINEAR AND POLYGONAL ANALYSES FOR CASE (6): 8-SEC SWELL FROM -60°, -40°, 60°

Spectral			Computed directions for triads <i>ijk</i> and pairs <i>ij</i>											
Line (k)	Period (sec)	Density (cm ²)	123	214	125	134	135	145	234	235	245	345	21	54
			Degrees											
122	8.39	0.00				-58	-58	-58	-57	-57	-57	-58		57
123	8.33	0.03				-59	-59	-59	-58	-58	-59	-59		58
124	8.26	14.20	I	I	I	-60	-60	-60	-60	-60	-60	-60	I	60
125*	8.19	84.03	n	n	n	-60	-60	-60	-60	-60	-60	-60	n	60
126	8.13	33.79	v	v	v	-60	-59	-60	-60	-60	-60	-60	v	60
127	8.06	22.62	a	a	a	-39	-39	-39	-39	-39	-39	-39	a	38
128*	8.00	152.17	l	l	l	-40	-40	-40	-40	-40	-40	-40	l	40
129	7.94	56.63	i	i	i	-39	-39	-39	-39	-39	-39	-39	i	40
130	7.88	12.93	d	d	d	60	61	61	64	62	61	62	d	59
131*	7.82	85.83				60	60	60	60	60	60	60		60
132	7.76	31.25				60	60	60	60	60	60	60		60
133	7.70	0.07				60	59	59	58	59	59	59		61
134	7.64	0.00				58	57	57	54	56	58	57		62

*Indicates spectral line closest to the frequency of the simulated wave.

TABLE 6. RESULTS OF LINEAR AND POLYGONAL ANALYSES FOR CASE (7): 8-SEC SWELL FROM -60° , -60° , -60°

Spectral			Computed directions for triads <i>ijk</i> and pairs <i>ij</i>											
Line (k)	Period (sec)	Density (cm ²)	123	214	125	134	135	145	234	235	245	345	21	54
			Degrees											
122	8.39	0.00				-60	-60	-60	-60	-60	-60	-60		59
123	8.33	0.00				-60	-60	-60	-60	-60	-60	-60		59
124	8.26	0.04	I	I	I	-60	-60	-60	-60	-60	-60	-60	I	60
125	8.19	14.58	n	n	n	-60	-60	-60	-60	-60	-60	-60	n	60
126*	8.13	80.56	v	v	v	-60	-60	-60	-60	-60	-60	-60	v	60
127	8.06	114.69	a	a	a	-60	-60	-60	-60	-60	-60	-60	a	60
128*	8.00	95.65	l	l	l	-60	-60	-60	-60	-60	-60	-60	l	60
129*	7.94	29.67	j	j	i	-60	-60	-60	-60	-60	-60	-60	i	60
130	7.88	28.18	d	d	d	-60	-60	-60	-60	-60	-60	-60	d	60
131	7.82	0.12				-60	-60	-60	-60	-60	-60	-60		60
132	7.76	0.01				-60	-60	-60	-60	-60	-60	-60		60
133	7.70	0.00				-60	-60	-60	-60	-60	-60	-60		61
134	7.64	0.00				-60	-60	-60	-60	-60	-60	-60		61

*Indicates spectral line closest to the frequency of the simulated wave.

TABLE 7. RESULTS OF LINEAR AND POLYGONAL ANALYSES FOR CASE (8): 8-SEC SWELL FROM -60° , -40° , 60°

Spectral			Computed directions for triads <i>ijk</i> and pairs <i>ij</i>											
Line (k)	Period (sec)	Density (cm ²)	123	214	125	134	135	145	234	235	245	345	21	54
			Degrees											
122	8.39	0.00				-52	-55	-51	-52	-51	-52	-52		52
123	8.33	0.00				-55	-57	-54	-54	-54	-54	-55		55
124	8.26	0.04	l	l	l	-58	-58	-57	-57	-56	-57	-57	I	57
125	8.19	14.54	n	n	n	-60	-60	-60	-60	-60	-60	-60	n	59
126*	8.13	80.82	v	v	v	-60	-60	-60	-60	-61	-60	-60	v	60
127	8.06	111.94	a	a	a	-50	-50	-50	-50	-50	-50	-50	l	51
128*	8.00	126.33	l	l	l	-53	-60	-49	-49	-46	-48	-52	i	44
129*	7.94	75.29	i	i	i	55	60	64	-58	-49	-62	-179	d	52
130	7.88	29.52	d	d	d	60	61	61	63	62	61	61		60
131	7.82	0.10				54	53	53	46	51	53	51		61
132	7.76	0.01				41	38	40	35	38	40	38		59
133	7.70	0.00				28	23	27	25	24	24	24		56
134	7.64	0.00				18	11	18	20	13	11	14		54

*Indicates spectral line closest to the frequency of the simulated wave.

with inclusion of random noise uniformly distributed in the range ± 2 cm to study the effect of expected white noise in field data. Directions determined in both runs were nearly identical except at spectral frequencies having negligible energy. Results of case (8) are similar to those of case (4).

In contrast, Esteva (1977), from more or less similar results, because of indiscriminate treatment of all triads, reported failure for cases (5)–(7). And since the array had been designed for periods between 25 and 7 sec, on the basis of additional simulation studies, erroneously concluded, that (1) gauge triads which are nearly equilateral have greater

direction discernability, (2) that the analysis gives correct directions only for waves coming from certain directions, and (3) that at the Pt Mugu site, directional information provided by the array adds little to the information that could be obtained from wave refraction studies, and thus seems hardly cost effective.

It may be remarked that his first conclusion, viz. that nearly equilateral triads have greater direction discernability is close to the truth, but only incidentally so, as the nearly equilateral triads, viz. 1-3-4, 3-4-5 and 3-2-5 all belong to the "YES" class, while the remaining seven triads belong to the "IF" and "NEVER" classes, which will give correct directions only by chance for class "NEVER" triads and are liable to error for class "IF" triads if precautions are not taken.

CONCLUSIONS

The approach based on phase/time/path difference concepts for the determination of the directional spectrum of waves from linear and polygonal arrays, has been shown to give consistently accurate results for simulated wave trains. It is shown how the criterion of Barber and Doyle (1956) should be used as a tool in designing arrays and in interpretation of directional information.

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