

VISCOUS DISSIPATION EFFECTS ON HEAT TRANSFER IN FLOW PAST A CONTINUOUS MOVING PLATE

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Abstract: The study of thermal boundary layer on taking into account the viscous dissipative heat, on a continuously moving semi-infinite flat plate is presented here. Similarity solutions are derived and the resulting equations are integrated numerically. It is observed that greater viscous dissipative heat causes a fall in the rate of heat transfer.

1. Introduction: Sakiadis (1961) studied in a series of papers the flow past a continuously moving plate in a stationary fluid environment. Both laminar and turbulent flows were considered by him. The heat transfer aspect of such a problem on a continuously moving flat plate was studied by Tsou, Sparrow and Goldstein (1967) without taking into account the viscous dissipative heat. As viscous dissipative heat plays an important role in such problems, it is now proposed to study the effects of viscous dissipative heat on the rate of heat transfer. In Sec. 2, the mathematical analysis is presented, followed by a discussion and in Sec. 3., the conclusions are set out,

Mathematical Analysis: Consider the flow of a viscous fluid past a moving sheet with the x -axis taken along the sheet, Fig. 1, and the y -axis is taken normal to the sheet.

If u and v are the velocity components along x and y -axis respectively, then under usual boundary layer approximation, the flow and temperature fields are governed by the following:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

Here T is the temperature of the fluid, μ the viscosity, ρ the density, c_p the specific heat at constant pressure, $\alpha = k/\rho c_p$ the thermal diffusivity and k is the thermal conductivity.

The boundary conditions are

$$u = U_0, T = T_w \text{ at } y = 0; u = 0, T = T_\infty \text{ as } y \rightarrow \infty \quad (4)$$

In equation (3), the second term on the right hand side represents the viscous

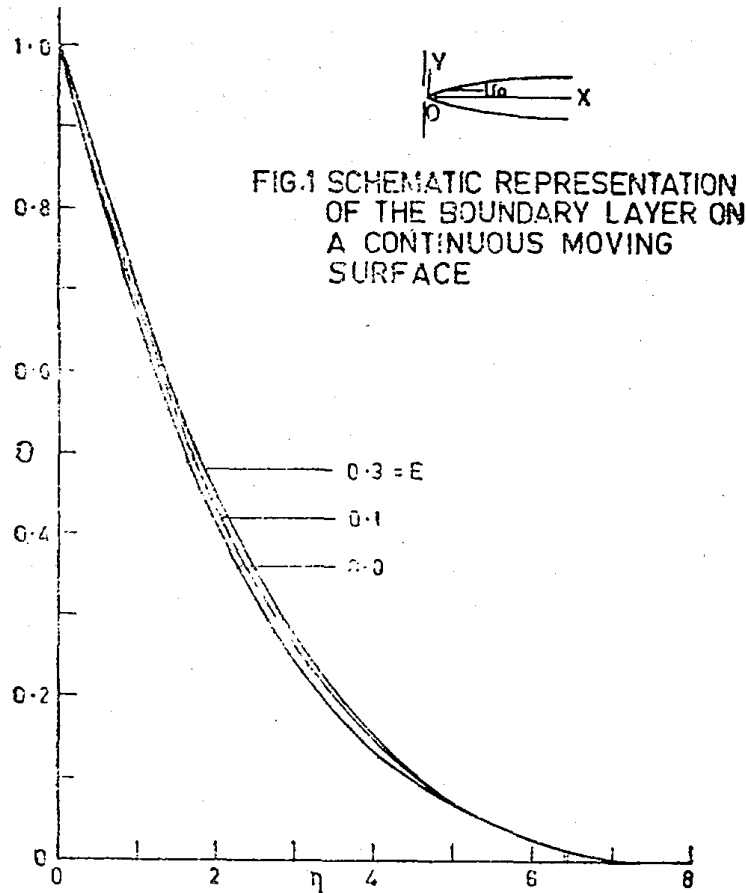


FIG.2 TEMPERATURE PROFILES, $Pr = 0.71$

dissipative heat. Also T_w is the temperature of the plate, T the temperature of the fluid far away from the plate, and U_0 is the velocity of the moving plate. On introducing the usual similarity transformations.

$$\eta = y \sqrt{\left(\frac{U_0}{\nu x}\right)}, u = U_0 f', v = \frac{1}{2} \sqrt{\left(\frac{U_0 \nu}{x}\right)} (\eta f' - f), \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad (5)$$

in equations (2) - (3), we get the following Blasius - type of boundary layer equations:

$$f'''' + \frac{1}{2} f f'' = 0 \quad (6)$$

$$\theta'' + \frac{1}{2} Pr f \theta' + Pr E f'^2 = 0 \quad (7)$$

where $Pr = \mu c_p / k$ (Prandtl number), $E = U_0^2 / c_p (T_w - T_\infty)$. The Eckert number and the boundary conditions are

$$f'(0) = 1, f(0) = 0, f'(\infty) = 0; \theta(0) = 1, \theta(\infty) = 0 \quad (8)$$

Equations (6) and (7) for $E = 0$ have been solved numerically in Tsou, Sparrow and Goldstein (1967). For incompressible fluids, the Eckert number is always small and hence we have integrated equations (6) and (7) for $E = 0, 0.1, 0.3$ and $Pr = 0.71$

numerically. The velocity profiles have been shown in Tsou, Sparrow and Goldstein (1967). We have shown the temperature profiles on Fig. 2. We conclude from this figure that greater viscous dissipative heat causes a rise in the temperature of the fluid.

The rate of heat transfer expressed in terms of the Nusselt number is given by

$$Nu/Re = \{-\theta'(0)\} \quad (9)$$

where

$$Nu = xQ/k(T_w - T_\infty)$$

The numerical values of $\{-\theta'(0)\}$ are entered in Table I. We conclude from these results that greater viscous dissipative heat leads to a fall in the rate of heat transfer.

Table I
Values of $\{-\theta'(0)\}$

Pr/E	0	0.1	0.3
0.71	0.3496	0.3332	0.3006

Conclusion: Greater viscous dissipative heat causes a rise in the temperature of the fluid but a fall in the rate of heat transfer.

Appendix—Numerical Method

“Shooting method” using Taylor series is employed. To satisfy the boundary conditions (8), three of which are given at $\eta = 0$ and the remaining two are given at $\eta = \infty$, we compute the solution of equations (6) and (7) assuming guessed values of $f''(0)$ and $\theta'(0)$. These solutions will in general not satisfy the boundary conditions $f'(\infty) = 0$ and $\theta(\infty) = 0$. Depending on whether the computed value of $f'(\infty)$ is less than zero or greater than zero, we either increase or decrease the guessed value assumed for $f''(0)$. This process is continued until $f'(\infty)$ tends to zero asymptotically. In a similar manner, $\theta'(0)$ is also corrected.

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